# Three Dimensional Geometry of the Heinike-Mikulicz Strictureplasty

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Three Dimensional Geometry of the Heinike-Mikulicz Strictureplasty

By

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Abstract:

Objective: The objective of this study is to assess the regional geometry of the Heineke Mikulicz (HM) strictureplasty.

Background: The HM intestinal strictureplasty is commonly performed for the treatment of stricturing Crohn's disease of the small intestine. This procedure shifts relatively normal proximal and distal tissue to the point of narrowing and thus increases the luminal diameter. The overall effect on the regional geometry of the HM strictureplasty, however, has not been previously described in detail.

Methods: HM strictureplasties were created in latex tubing and cast with an epoxy resin. The resultant casts of the lumens were then imaged using a computerized tomography. Utilizing three-dimensional vascular reconstruction software, the cross-sectional areas were determined and the surface geometry was examined.

Results: The HM strictureplasty, while increasing the lumen at the point of the stricture, also results in a counterproductive luminal narrowing proximal and distal to the strictureplasty. Within the model used, cross-sectional area was diminished 25 to 50% below baseline. This effect is enhanced when two strictureplasties are placed in close proximity to each other.

Conclusions: The HM strictureplasty results in alterations in the regional geometry that may result in a compromise of the lumen proximal and distal to the location of the strictureplasty. When two HM strictureplasties are created in close proximity to each other, care should be undertaken to assure that the lumen of the intervening segment is adequate.
**Introduction:**

Crohn's disease is a chronic intestinal inflammatory condition of unknown etiology that typically affects the small intestine and colon. Inflammation from small bowel Crohn's disease is often multi-focal in nature and can give rise to multiple non-adjacent areas of localized intestinal stricturing. Patients with chronic obstructive symptoms from multifocal strictures are often treated with intestinal strictureplasty. The most commonly applied strictureplasty technique is the Heineke Mikulicz (HM) strictureplasty.¹ This procedure is named after the pyloroplasty technique from which it is derived. The procedure involves creating an anti-mesenteric incision along the long axis of the intestine.²(Figure 1) This incision is centered over the focal area of narrowing and is extended into relatively normal tissue both proximal and distal to the stricture. This longitudinal incision is then closed in a transverse fashion. With this, the HM strictureplasty draws tissue from areas proximal and distal to the stricture to add to the circumference at the stricture site. Because tissue proximal and distal to the site are shifted, changes in the regional geometry away from the stricture itself are likely. While it is widely recognized that the HM strictureplasty is effective at relieving the narrowing of a Crohn’s stricture, a detailed analysis of the effects on the regional geometry of the HM strictureplasty has yet to be described. The following study was undertaken to provide an experimental model in which to study the geometry of intestinal strictureplasties and to elucidate the geometric character and regional effects of the HM strictureplasty.

**Methods**

To precisely and accurately study the geometry of a single HM strictureplasty as well as the interaction between pairs of strictureplasties, we designed a robust elastic model. Latex tubing with 2 cm inner diameter and 0.15 cm wall thickness (McMaster-Carr, Elmhurst, IL) was used to model the baseline cylindrical intestinal geometry. The HM procedure was carried out on
the tubing: linear incisions were made along the long cylindrical axis, closed transversely using interrupted 4-0 Polysorb suture (Tyco, Norwalk, CT) taking care that the incision vertices were well approximated. The suture lines were made water tight by external application of silicone rubber 732 multi-purpose sealant (Dow Corning, Midland, MI). The post-procedure tubing was cast with EpoFix cold setting embedding epoxy resin (Electron Microscopy Sciences, Hartfield, PA). EpoFix requires no heating thus assuring the rubber mold would not deform during the casting process. A setting time of 36-48 hours was used at room temperature. To obtain geometric parameters, the epoxy casts were imaged with computerized tomography in a Phillips iCT256 scanner using 0.9mm slice thickness, 0.045mm slice increments, V=120kV, I=37mA, mAs=30mAs. The digitalized imaging allowed for calculation and analysis of the cross-sectional area by commercial Philips 3D vascular re-construction software (See Figure, Supplemental Digital Content 1A, which demonstrates the digital reconstruction of the strictureplasty) (See Video, Supplemental Digital Content 2, which demonstrates three dimensional reconstruction of a HM strictureplasty) (See Video, Supplemental Digital Content 3, which demonstrates three-dimensional reconstruction of two HM strictureplasties in series) (See Video, Supplemental Digital Content 4, which demonstrates three-dimensional reconstruction of a Michelassi strictureplasty). The epoxy casts were also imaged using a Nikon DX90 camera. The camera was controlled externally by the Nikon Capture Control software. All data analyses were performed using Matlab (Mathworks, Natick, MA).

For the single strictureplasties, three enterotomy lengths were studied: 2, 3, and 4 cm. For the double strictureplasty models, the procedure was the same except that two linear enterotomies were initially created (2 and 3 cm enterotomies were studied). Care was taken so that the two incisions were along the same longitudinal axis. Furthermore, the separation between two strictureplasties was measured as the shortest linear distance between the incisions.
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Advanced mathematical analysis of the resultant geometry of the HM strictureplasty was undertaken by considering the outer most lumen layer as an idealized two dimensional surface and studying its intrinsic geometry. The intrinsic geometry of a surface consists of the collection of all distances between points as measured on the surface. The connection between the intrinsic geometry of a surface and the configuration it assumes in space is not trivial and is provided by the Gaussian curvature. (See Text, Supplemental Digital Content 5, which provides a detailed description of the mathematical principles)

In general, when an initially flat (or cylindrical) surface is cut and reconnected along straight lines the resulting intrinsic geometry remains flat almost everywhere and contains only conical defects in which Gaussian curvature is condensed in points. The magnitude of a Gaussian curvature condensation can be identified with the opposite of the angle excess in the cone. A birthday-hat like structure, which is constructed by cutting out a wedge of head angle $\alpha$ from a disc and reconnecting the free edges of the disc, corresponds to a positive Gaussian curvature condensation of magnitude $\alpha$ at the vertex (See Figure, Supplemental Digital Content 6A, which demonstrates the described curvature). Similarly if a wedge of head angle $\beta$ is now connected to the cut sides then the resulting Gaussian curvature condensation is of magnitude $\alpha - \beta$ (See Figure, Supplemental Digital Content 6B & C, which demonstrates the described curvature). It is important to emphasize that the condensation of Gaussian curvature to a point does not mean that the geometry was changed only at a point; in the above examples a whole
wedge of material was either introduced or eliminated in order to generate the desired Gaussian curvature condensation.

**Results:**

In this model, the HM strictureplasty, depending on the length of the enterotomy, increases the luminal cross-sectional area by 50 to 150% above the baseline lumen at the point of the strictureplasty (Figure 2). Regional distortions from the strictureplasty, however, generated a decrease in luminal cross-sectional area of 25 to 50% below baseline just proximal and distal to the strictureplasty site. This compromise of the residual lumen was dramatically increased when two strictureplasties were placed in close proximity of each other (Figure 3).

For a detailed analysis of the geometry of the HM strictureplasty we began with the model of the single strictureplasty in isolation (Figure 2A). We identify three locations of Gaussian curvature condensation. First, there is the small central region in which the enterotomy end-points (a and a’ in Figure 1) are sutured together. In our physical models, this corresponds to the midpoint along the suture line (black open circle in Figure 2A bottom panel). Geometrically, the strictureplasty procedure in this point corresponds to taking two full circles and joining them along a radial cut as shown in Figure 4A, where the centers of the circles map onto points a and a’, and the joined centers give rise to a surface geometry similar to that seen in Figure 4B. This central point carries a negative Gaussian curvature condensation of $-2\pi$ (saddle-like structure). The ridges of the saddle are separated by four valleys (shaded in yellow in Figure 2A) and also clearly seen in different projections of the mathematical models in Figures 4D-F.

A second set of curvature condensation points flank the central $-2\pi$ region. These points arise from the transverse closure of the enterotomy at the ends of the suture line (b and b’ in Figure 1 and red open circles in Figure 2A). Geometrically, the structure of each of these
flanking corners corresponds to connecting the two radial lines in a semi-circle to generate a cone of Gaussian curvature condensation of \( +\pi \). The fact that the Gaussian curvature condensations sum up to zero (two of \( +\pi \) and one of \( -2\pi \)) is not coincidental and is associated with the fact that far from the strictureplasty-site the geometry remains unchanged. Further details of these concepts are contained within the supplemental text (See Text, Supplemental Digital Content 5). In summary, the geometry of the HM strictureplasty is set by the linear enterotomy and transverse closure that generates three points of curvature condensation: a central saddle like structure and two flanking cones.

The observed geometry of the single strictureplasty is strongly dominated by the \(-2\pi\) Gaussian curvature condensation. Within the model, cutting out the points of positive Gaussian curvature condensations, as well as introducing various cuts to the anti-mesenteric side does not alter the geometry significantly. This conclusion is also supported by Figure 4 where an elastic model of the negative Gaussian curvature condensation alone (excluding both the cylindrical geometry and the positive Gaussian curvature condensation) (Figure 4B), successfully reproduces the shape of the corresponding region in the HM strictureplasty as it appears in Figure 4C. (Also See Video, Supplemental Digital Content 7, which demonstrates how the model successfully reproduces the shape of a HM strictureplasty).

We next investigate what impact this geometry has on luminal cross-sectional area of our models. Figure 2B presents the CT derived cross-sectional areas \( A \) as a function of curved distance along the post-strictureplasty model, i.e. arc length \( \ell \). We center our data along the transverse suture line and define this plane as \( \ell = 0 \); furthermore, we use the un-deformed tube diameter \( d \) as our internal length scale. In the case of all three enterotomy lengths (2, 3, and 4 cm), two regimes are immediately definable. Regime 1, away from the enterotomy, and defined by \( \ell \leq -d \) or \( +d \leq \ell \), (outside the green box in Figure 2B) where \( A \) is equal to \( A_0 \) the area of
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the un-deformed cylinder, and the corresponding relative change in cross-sectional area

\((A - A_0)/A_0\) is zero. Significant deformation fields induced by the strictureplasty geometry are

confined to the vicinity of the enterotomy extending one tube diameter in each direction,

\(-d \leq \ell \leq +d\). We define this as regime II, (inside the green box in Figure 2B). In this region both

area dilation and area contracture are observed. Centrally located under the suture line is an area

of strong dilation: \(A(\ell = 0) \approx (1.5 - 3) \cdot A_0\). However this dilated region does not smoothly

connect to the un-deformed cylinder of regime I, but rather is flanked by regions of area

contracture both distally and proximally: \(A(\ell = \pm d/2) \approx 1/2 \cdot A_0\). It is these flanking regions that

make the re-connection to the un-deformed cylinder. The existence of strong dilation is not

surprising as the procedure is successful in dilating strictured bowel. However, the luminal

compromise within the flanking regions has not been previously described.

We note that the existence of both dilated and contracted regions within the

strictureplasty is consistent with its saddle-like geometry discussed above. Figure 4 shows the

generic shapes generated by fussing two circles in different projections. The dilated region

exists directly under the suture line and corresponds to the region around the horizontal ridge

where the circles were sutured together. The size of the area underneath the ridge clearly depends

on its length, which is simply the length of the initial enterotomy in our models. Indeed, the data

in Figure 2B show that the dilation increases with enterotomy length. Moreover, Figure 4E and

4F clearly show that just proximal and distal to the horizontal suture line, the sheet is pinched

inwards. The four valleys radiating from the midpoint of the suture line (\(\sim 2\pi\) condensation

point) drive this inward displacement. In fact, by comparing the cross-sectional images of our

model strictureplasties in the contracture area (see Figure 2B bottom-most images), it is easily

appreciated that the structure is nearly triangular in agreement with the triangular opening seen in

Figure 4F. Geometrically, the degree of pinch-off is independent of circle radius or the length of
the suture line, as long as there is sufficient length for the tube to close on the mesenteric side.

Again, our data are in agreement, showing that the degree of contracture is less sensitive to enterotomy size than the degree of dilation. In summary, we conclude that the dilation (50-150% increase in cross-sectional area) is simply driven by the transverse closure of the enterotomy; however, the strong condensation of negative Gaussian curvature that occurs during this closure induces regions of area compromise (~25-50% decrease relative to un-deformed tubing).

The second part of our study focuses on how the global geometry and luminal area change as multiple strictureplasties are placed in series. Figure 5 shows images for a set of 2 cm and 3 cm enterotomies with enterotomy separation varying from 1 cm to 7 cm (at 1 cm intervals). Visually it is apparent that below a separation of 3 cm, a transition occurs. To more precisely characterize this transition, we follow the cross-sectional area of the different models using CT. The data in Figure 3 can be divided into two regimes. A weak-interaction regime for $\lambda > d$, where $\lambda$ is the strictureplasty separation length (See supplemental Digital Content 1B for further definition). In this regime, the two strictureplasties have the same geometry as in the single enterotomy cases studied above: central area of dilation flanked by areas of contracture. Beyond each strictureplasty, the cross-sectional area returns to that of the un-deformed cylinder: $A(\ell = 0) \approx A(\ell = \pm \infty)$. The conclusion here is that if separated by at-least one tube diameter, the geometry of multiple strictureplasties is independent of one another. A rather dramatic transition occurs once the enterotomies are placed within one tube diameter: $\lambda \leq d$. Within this strong-interaction regime, the two strictureplasties strongly interact causing a very severe collapse of the cross-sectional area between the two sites: $A(\ell = 0) \approx 0$. The nearly total collapse of the inter-strictureplasty area is far beyond the milder contracture encountered with single strictureplasties, where the decrease in area was on the order of 25-50% versus over 85% encountered when two strictureplasties are placed in close proximity. (See Video, Supplemental
Digital Content 8, which demonstrates a virtual fly-through of the lumen of two strictureplasties placed in proximity)

As detailed in the introduction, multiple strictures can surgically be treated with either several HM procedures or alternatively with the Michelassi iso-peristaltic strictureplasty. We studied one model of the Michelassi (Figure 6). Our data show that the procedure leads to a nearly four-fold increase in luminal area, consistent with the doubling of the luminal diameter. Furthermore, the beveling effect at the end-points releases some of the Gaussian curvature condensation. This likely plays a role in alleviating any proximal or distal contracture that would otherwise occur. Unlike the circle-to-circle geometry that is inherent in the Heineke-Mikulicz, the wedges presented in Supplemental Digital Content Figure 6B and 6C capture the Michelassi end-point geometry more clearly. Note that some curvature condensation still occurs, however it is decreased by the amount of angle removed during wedge creation.

Lastly, we carried out experiments on tubes of different thickness and diameters to better understand the above observed luminal collapse between two strictureplasties. A phase diagram of double enterotomy/strictureplasties as a function of tube thickness ($t$), tube diameter ($d$), enterotomy length, and strictureplasty separation distance ($\lambda$) is given in Supplemental Digital Content 9 (See Figure, Supplemental Digital Content 9, a phase diagram of double strictureplasties as a function of tube thickness, diameter, enterotomy length, and strictureplasty separation). Briefly, three dimensionless parameters are defined: $\psi = \lambda/d$, $\alpha = \text{tube thickness/enterotomy length}$, and $\phi = \text{enterotomy length}/d$. And our data show that the criteria for placing two HM strictureplasties within the *strong-interaction regime* are $\psi < 1$, $\phi \geq 1$, and $\alpha \leq 0.1$.

*Discussion:*
With our model for intestinal strictureplasty we found a 25-50% decrease in the normal residual cross-sectional area just proximal and distal to an isolated HM strictureplasty. For the strictureplasties performed in isolation, the length of the enterotomy correlated with the cross-sectional area at the point of the strictureplasty. Larger enterotomies resulted in greater increases in the cross-sectional area at the point of the strictureplasty, while the degree of luminal compromise proximal and distal to the strictureplasty was less dependent upon the length of the enterotomy. That is to say, increasing the length of the enterotomy for a strictureplasty performed in isolation results in a significant increase in the lumen at the point of the stricture without much in the way of an increase in the compromise of the lumen proximally and distally.

When two strictureplasties are created in close proximity to each other, the compromising affect on the lumen is dramatically increased. This additive effect becomes prominent when the strictureplasties are positioned within a distance equal to or less than the diameter of the normal un-distorted lumen. Given that the HM strictureplasty is designed to increase luminal diameter at the point of stricturing by shifting tissues normally located above and below the stricture, some degree of narrowing of the lumen proximally and distally could have been anticipated. Yet, the degree to which this happens, at least with our model, was surprising. The effect seen when two strictureplasties are placed in close proximity is dramatic.

It is important to note that the distance between strictureplasties is different from the distance between the strictures themselves. The distance between strictureplasties is a function of the distance between the strictures and the length of the enterotomy that is utilized to create the strictureplasty. For example, if two focal strictures located 7 cm apart in a segment of intestine with a baseline diameter of 3 cm are managed with HM strictureplasties each performed with a 4 cm enterotomy, the resultant strictureplasties would be 3 cm apart and thus within the range where significant luminal compromise of the segment between the strictureplasties may
occur. Increasing the length of enterotomy for strictureplasties placed in series will shorten the distance between the strictureplasties themselves and thus potentially increase the collapse generated by the interaction between the two strictureplasties. In other words making enterotomies that are excessively long will result in strictureplasties that ultimately will lie closer to each other and hence excessively long enterotomies may have a counterproductive effect that decreases the lumen in between the two strictureplasties. On the other hand, as mentioned above, shortening the enterotomy for a HM strictureplasty in isolation will not significantly affect the degree of narrowing proximally and distally to the strictureplasty. Either way decreasing the length of the enterotomy will decrease the expansion of the lumen at the stricture sites.

The safety and effectiveness of HM strictureplasties in the management of stricturing Crohn's disease of the small intestine has been well-established.1,5-8 The technique is an effective means of alleviating the symptoms of chronic partial obstruction while at the same time preserving intestinal length and functional absorptive surface area. From the excellent short term results reported in multiple case series, it is reasonable to conclude that any luminal compromising that may occur in proximity to the HM strictureplasty is not likely to result in postoperative obstructive symptoms in the short term. The long-term consequences on luminal narrowing, however, may prove to be more troubling. Much attention has been paid to the consequences that the post surgical residual intestinal lumen after intestinal anastomosis may have on the recurrence rates for Crohn's disease. Considerable literature has been devoted to how varying anastomotic techniques would affect the likelihood of recurrence.9-13 So far no such consideration has been applied to strictureplasty techniques. It has been suggested that anastomotic techniques that result in diminished or compromised luminal cross-sectional areas may result in earlier recurrences of inflammation and/or symptoms from recurrent Crohn's
Some have contended that stasis of luminal contents may generate or aggravate the inflammatory response. Clinical observations have also suggested that alleviation of stasis may result in improvement in disease activity. Even if residual lumen size were to have no effect on the activity of inflammation it would seem possible that an already compromised lumen would more readily constrict to a critical diameter that leads to earlier development of obstructive symptoms.

It is interesting to note that some authors have noted that upon re-exploration for recurrent Crohn's disease in those patients who had undergone previous intestinal strictureplasty, the strictureplasty sites themselves are often free of recurrence. Recurrences, however, are commonly noted to be in the same general region of the initially treated disease. In other words, recurrences may not typically occur at the strictureplasty site, but rather in the regions proximal or distal to the previous site. It is also been reported that recurrences are higher when multiple strictureplasties are employed when compared to strictureplasties performed in isolation.

Surgeons with experience in treating Crohn's disease have advised against placing HM strictureplasties in close proximity. These recommendations are based upon concerns regarding tension on the suture lines and possible compromise of the blood flow to the tissues in between suture lines that are placed in close proximity. We believe this study provides additional reasons for concern when performing HM strictureplasties that are separated from each other by relatively short distances. Under such circumstances it may be advisable to use alternative techniques such as resection, the Finney strictureplasty, or the Michelassi strictureplasty. The Michelassi strictureplasty appears to be well suited for managing multiple strictures located in close proximity. Our model demonstrated that this technique resulted in dramatic increase in the lumen throughout the length of the strictureplasty without any significant compromise of the natural lumen on either the proximal or distal ends.
The detailed mathematical analysis of the three-dimensional geometry of the HM strictureplasty is both complex and challenging. Such detailed analysis, however, is more than a theoretical endeavor. Through such analysis and modeling it may be possible to propose modifications to the surgical technique that could ameliorate the effects described, hence, the detailed mathematical analysis is included in this manuscript.

The main limitation of this study is that the model is created from inanimate materials and thus it cannot compensate or predict the variables that may occur in living tissue, such as motility, variable compliance, remodeling, and tissue growth compensation. The model, however, does provide for the consistency and reproducibility necessary for accurate measurement and analysis. Another limitation of this study is that all the experiments were performed in tubes without focal stricturing. This was done for the sake of consistency, but the absence of a focal stricture should not affect the key observations made. Because the surgical procedure extends both proximally and distally beyond the strictured area the global hyperbolic geometry caused by negative Gaussian curvature condensation at the central point will be dominated by tissue properties at the ends of the enterotomy. These ends exist in normal tissue. While the stricture tissue could potentially impact how the flanking conical structures develop, these areas are not relevant to the geometric issues that are the focal point of this study.

In summary, our model suggests that the HM strictureplasty results in compromise of the lumen proximal and distal to the strictureplasty site. This effect is greatly increased when to strictureplasties are placed in close proximity to each other. Care should be undertaken when performing multiple HM strictureplasties to assure that the intervening lumen is adequate.
Acknowledgments:

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References:


Figures:

Figure 1:
Heineke-Mikulicz strictureplasty procedure performed in a patient with focal stricturing. (A.) Linear incision is made along the anti-mesenteric border, extending proximally (a) and distally (a') across the stricture into healthy bowel. (B. and C.) The incision is closed transversely with the approximation of vertex points a and a', which initially were separated by the length of the incision. (D.) Completed Heineke-Mikulicz strictureplasty.
Figure 2: Models of single enterotomy strictureplasties of varying length. (A.) CT derived three-dimensional reconstructions of final Heineke-Mikulicz geometries generated from 2, 3, and 4cm linear enterotomies. The shading in the last set of images highlights the different geometric structures discussed in the main text. (B.) shows the relative cross-sectional areas \((A - A_0)/A_0\), where \(A_0\) is the area of the un-deformed tube) of the three models from distal to proximal ends across the strictureplasty sites as a function of arc length \(\ell\). Two regimes are identified: \(I.\) \(+d \leq \ell\) and \(-d \geq \ell\) (outside the green box) where \(A\) is equal to that of the un-deformed cylinder and \(II.\) \(-d \leq \ell \leq +d\) (inside the green box) where \(A\) deviates strongly and represents the region most strongly affected by the HM procedure, in both cases \(d = 20\)mm and is the diameter of the un-deformed tube. Regime \(II.\) can further be subdivided into a central area of strong dilation, where \(A(\ell = 0) \approx (1.5 - 3) \cdot A_0\) and the degree of dilation increases proportionally with increasing enterotomy length, and flanking areas of contraction just proximal and distal to the point of dilation, where \(A(\ell = \pm d/2) \approx 1/2 \cdot A_0\) and the degree of narrowing dependents less strongly on enterotomy length.
Figure 3: (A.) CT derived cross-sectional areas of model double strictureplasties as a function of enterotomy length, 2cm (red data) and 3cm (blue data), and strictureplasty separation (1 to 7cm). The curves are offset for each separation distance and centered at the mid-point for clearer visualization. The baseline in each set is drawn in as the dashed gray horizontal line and corresponds to a cross-sectional area identical to the un-deformed tube \( A_0 \) (solid black bar corresponds to a 100% change in cross-sectional area). Two regimes are identified: a weak-interaction regime occurring for separation distances greater than one tube diameter (white background data in (A.) and representative set with CT cross-sectional images in (B.)) and a strong interaction regime when strictureplasties are placed within one tube diameter (yellow background data in (A.) and representative set with CT cross-sectional images in (C.)). Within the weak-interaction regime, the two strictureplasties sites have the same local structure as that of the single strictureplasties in figure 2: central dilation with flanking contractions; however, beyond each strictureplasty the cross-sectional area returns to baseline: \( A(\ell = 0) \approx A_0 \). Within the strong-interaction regime, the two strictureplasties strongly interact causing a very severe collapse of the cross-sectional area between the two sites: \( A(\ell = 0) \approx 0 \).
Figure 4: The geometry of a single HM strictureplasty is dominated by the $-2\pi$ Gaussian curvature condensation at the center of the strictureplasty site. (A.) The enterotomy ends, marked $a$ and $a'$ in Fig.1 can be considered as the centers of two identical circles whose radii are half the enterotomy length. Within this framework it is obvious how the suturing of the two circles one to the other generates a $2\pi$ angle excess which corresponds to a $-2\pi$ Gaussian curvature condensation. (B.) The configuration obtained by minimizing the elastic bending energy of the two connected discs forming a $-2\pi$ Gaussian curvature condensation. The suture lines are assumed to have no bending rigidity, and the discs are not allowed to self-intersect. (C.) The scanned 3cm enterotomy length model (shown in Fig 2A) with a region of distance < 1.5cm around the central vertex marked in dark blue. See supplementary material for a 3D movie. (D.) Top, (E.) diagonal and (F.) side views of the elastic bending minimizing configuration. The triangular opening visible in (F.) accounts for the proximal and distal cross-section area decrease visible in Fig2 B.
Figure 5: Cast models of two strictureplasties (2cm and 3cm enterotomies) separated by 2, 3, 4, 5, 6, or 7cm. The inter-plasty separation is measured as the distance between the inner vertices of the two strictureplasties in the un-deformed tubes (see supplemental figure 1). The images clearly show that as two strictureplasties approach within one tube diameter (2cm), a strong change in global geometry occurs.
Figure 6: Cross-sectional area data for a model Michelassi strictureplasty (A.) and the CT three-dimensional reconstruction of the model (B.). The data show central dilation by a factor of 4 over the un-deformed tube consistent with radius doubling in the reconstructed section. Importantly, the connection between the dilated region and the un-deformed (region indicated by black arrows) is smooth with no proximal or distal contracture as in the Heineke-Mikulicz.