

Pulled Fronts and the Reproductive Individual

Fluctuation-regularized Front Propagation Dynamics
Elisheva Cohen, David A. Kessler and Herbert Levine
arXiv.org/cond-mat/0406336

An exactly soluble noisy traveling wave equation appearing in the problem of directed polymers in a random medium
Éric Brunet and Bernard Derrida
arXiv.org/cond-mat/0409261.

Recommended with a commentary by Leo P. Kadanoff, University of Chicago.

In “War and Peace” L. Tolstoy expressed his opinions about the relative importance of “great men” and the overall force of events in determining the outcome of historical processes. He said that particular individuals are relatively unimportant in comparison to the overall “forces of history”. Amusing enough, the relative importance of the individual and the mass has become an interesting and important distinction in the development of theories related to front propagation in condensed matter systems. Two recent papers focus upon situations in which the individual may count for a lot.

The problem is one of front propagation in which a stable phase propagates into the region previously occupied by a less stable one, or a more robust population occupies territory previously occupied by a less “fit” population. As pointed out in detail by an extensive recent review¹, there are two possibilities for the propagation of a one-dimensional front: Either the motion is “pulled” by the little bits of stable phase extending far into the

1. Wim van Saarloos, “Front Propagation in Unstable States” *Physics Reports* **386** 29-222 (2003).

unstable region or alternatively it is “pushed” by the growth which occurs within the bulk of the the boundary region. If the propagating phase were a population of humans we would say the growth was pulled by the individuals at the very edge of the frontier or, conversely, that it was pushed by population pressure in the bulk.

Naturally, the distinction between the two cases corresponds to a difference in behavior. The pulled case is much more delicate, subject to fluctuations, and responsive to small effects at the frontier. Both papers deal with the pulled case, and emphasize this delicacy. In the classic pulled situation, that of the equations first described by R. A. Fisher² and A. Kolmogorov and coworkers³, the growth rate approaches its maximum, a constant value, at the frontier. Cohen, Kessler, and Levine consider in contrast a situation in which the growth rate grows linearly with distance.⁴ Then, in a continuum model, the front velocity continually accelerates. If we have instead a discrete model in which the population contains at population saturation N individuals per unit of length, the front velocity depends upon N and is proportional to $(\ln N)$ if the spatial behavior is driven by a second order difference, as in the preprint, or $(\ln N)^{1/3}$ if there is a second derivative (see ref 4). This extreme sensitivity to graininess is characteristic of pulled models. Even in the F-KPP model of refs 2 and 3, the velocity contains a correction proportional to $1/(\ln N)^2$. In contrast, in pushed models the characteristic behavior of the

² R. A. Fisher, The Wave of Advance of Advantageous Genes, *Annals of Eugenics* 7, 355-369 (1937).

³ A. Kolmogorov, I. Petrovsky, and N. Piscounov, *Moscou Univ. Bull. Math. A* 1, 1 (1937).

⁴ See also the earlier papers L. S. Tsimring, H. Levine and D. A. Kessler, *Phys. Rev. Lett.* 76, 4440 (1996); D. Ridgway, H. Levine, and L. Tsimring, *J. Stat. Phys.* 87, 519 (1997) which use a spacial continuum model. In contrast the paper described here uses a discrete model in space.

velocity is to have a much weaker graininess correction, one proportional to some inverse power of N .

The importance of the individual on the frontier is shown even more strikingly in the paper of Brunet and Derrida. Here there is once more a pulled front and a discretization involving N particles per unit of length. Now the particles exist in a noisy environment with a noise of order $N^{-1/2}$. The main result of the paper is that the front velocity is determined by the noise on the last particles, standing at the far frontier⁵. Indeed, sometimes the individual does really matter.⁶ Of course there are other situations, other models, in which the fronts are “pushed” and well placed individuals count for less. Dynamical systems can do lots of different things. It all depends.

⁵ in numerical simulations published in 2001 on a discrete spatial model E. Brunet, B. Derrida, Effect of Microscopic Noise on Front Propagation J. Stat. Phys. 103, 269-282 (2001) show numerically that the velocity and the diffusion constant are not modified if the noise is limited to the first site where the front is non-zero.

⁶ Sykes, Bryan, “The Seven Daughters of Eve”, W. W. Norton (2001). In this book Sykes argues that virtually all of the 560 million modern Europeans descended from seven individual women who lived tens of thousands of years ago.