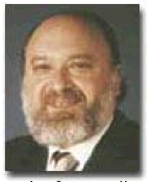
Reference Frame

The Unreasonable Effectiveness of . . .

Leo P. Kadanoff



Science is full of wonderful ideas and discoveries. Let me tell you about some exciting things I've heard about in the past year.

. . . A little leak

Two geologists at the Lamont?Doherty Geological Observatory, William Ryan and Walter Pitman, have proposed an audacious but apparently correct scenario for the early history of the Black Sea in their recent book, *Noah's Flood: The New Scientific Discoveries about the Event that Changed History* . They suggest that 7600 years ago, the Black Sea came to the end of its life as an isolated freshwater lake. At that time, it was comparable in size to Lake Michigan. Previously, a substantial warming had occurred. Evaporation lowered the Black Sea surface in comparison to the saltwater of the nearby Aegean and Mediterranean. The latter were rising with the oceans as they all shared water freed by glacial melts. These waters were held in place, away from the Black Sea, by a land mass including the present area of Istanbul.

Then the land barrier sprung a leak. Saltwater trickled into the Black Sea, dug a channel, and soon became a torrent equivalent to a thousand Niagaras. The water level in the Black Sea rose steadily. After a year or two, the waters had risen by 100 meters, and the freshwater lake had become a much larger saltwater sea, connected to the Aegean by the channel we call the Bosporus.

There are many stories here. One is about the amazing range of scientific knowledge and tools the authors used to establish their conclusions. Sonar established the basic form of the sea bottom. Drilling and underwater surveying were used to obtain sea floor samples, containing many finely ground seashells, indicative of a sea bottom. The lack of beachlike formations suggested rapidly rising waters. Carbon-14 gave dates. Biological knowledge of aquatic species showed a very rapid turnover from freshwater to saltwater. And overall knowledge of geological history and geomorphology provided the structure that permitted the confident analysis of this event. Most recently, Robert Ballard and coworkers have used an underwater robot to turn up apparent evidence of human habitation below the Black Sea. Could this discovery be connected with the flooding?

The event itself is another story. It was big. Imagine that over a period of two years, Lake Michigan rose by 100 meters. It would come up to the 30th story of the Sears Tower, covering most of Illinois with saltwater. Of course, exactly this thing cannot happen here and now. But other big things can. Maybe the worriers about global warming have a point.

. . . Wigner and Dyson

This year, the University of Chicago saw an odd coincidence: Two similarly named mathematical scientists, Persi Diaconis and Percy Deift, came to give distinguished lecture series--one in statistics, the other in mathematics. They each talked about random matrix theory and its relation to other topics in physics and mathematics, including quantum gravity, the Riemann zeta function, and integrable systems.

Some explanations are in order. This zeta function is one of the central elements of pure mathematics. It is defined in terms of prime numbers, and it has an infinite number of zeros arrayed along a line in the complex plane. There are formulas that enable one to translate statistical information about the spacing of zeros into knowledge about the spacing of prime numbers, and vice versa.

The random matrix idea grew out of Eugene Wigner's investigations of the highly excited states of nuclei. A nucleus is too complex for us to even give an accurate representation of its Hamiltonian. In 1951, Wigner hypothesized that different nuclei might behave as if the Hamiltonian of each one was a random matrix, with each matrix having been picked from an ensemble determined by the Hamiltonian's symmetry properties. Freeman Dyson noticed that there are three such ensembles that make especial physical and mathematical sense. The statistical distributions of energy levels generated by these ensembles show highly structured spectra with a considerable tendency for neighboring levels to repel one another and spread out as much as they can. The statistical properties of any stretch of a few tens or hundreds of consecutive highly excited levels tends to be universal, that is, independent of everything but the symmetry of the Hamiltonian and the average level spacing.

same statistical distribution could be used to describe the spacing of zeros of the zeta function. Andrey Nickolayevich Kolmogorov proposed a roughly similar kind of universality for the fine structure of turbulence problems. Since these great works, universality has been sort of expected in hydrodynamics and statistical physics problems.

Peter Sarnack and Zeev Rudnick constructed theorems that support Dyson's ideas about the relation between the distributions from the zeta function and from the matrices. Diaconis, in one of his Chicago lectures, described how he tested the hypothesis of the identity of these distributions using some elegant applications of modern statistical analysis. These numerical tests, which followed up on earlier work by Hugh Montgomery and Michael Berry, show the robustness and power of the connection proposed by Dyson.

But there is more to this universality. Not only is the Wigner/Dyson answer applicable to almost all matrices, but it applies to many other problems as well, including quantum chaos and universal conductance fluctuations. Moreover, random matrices have eigenvalues whose distribution is given by a statistical mechanics problem involving particles with Coulomb-like interactions. These problems fit into the general class of exactly solvable statistical mechanics models, including, for example, the two-dimensional Ising model and two-dimensional quantum gravity. These models can, in turn, be converted into problems in Hamiltonian dynamics by a trick equivalent to Feynman's path-integral formulation of quantum mechanics.

More yet: In his lectures, Deift pointed out that the random matrix problem has a natural connection to problems of integrable systems. He discussed "the remarkable discovery of Gardner, Greene, Kruskal and Miura in 1967 of a way to integrate the Korteweg?deVries equation, a particular infinite dimensional Hamiltonian system arising in the theory of water waves." Amazingly, this work gives a method for finding the complete spacetime solution of this nonlinear partial differential equation. In hindsight, we see that the exact solutions can also be found by constructing all the different integrals of the motion, or by developing a description of the independent modes of excitation of the system (as in the seminal 1965 work of Martin Kruskal and Norman Zabusky on solitons), or by several other apparently unconnected methods. Many different dynamical problems have been solved by these methods. The solution of these problems is one of the great achievements of the mathematical and physical sciences in the second half of the 20th century. According to Deift, all these integrable systems fit naturally into the same skein of ideas as problems in random matrix theory, statistical physics, quantum gravity, function theory, and many more areas.

Edouard Brézin and Anthony Zee have used renormalization arguments to understand the random matrix and Hamiltonian universalities. They describe these problems by indicating their relation to the exactly solvable "Gaussian model" of statistical mechanics. They explain universality as a consequence of the robustness of Gaussian probability distributions under changes of variables. Even though we do understand many aspects of this universality, one can marvel that so many different problem areas are all interconnected in this wonderful and apparently magic fashion. This "magic" signals that there are probably additional deep connections, yet to be discovered.

Coming to a point

My colleagues Tom Witten, Shankar Venkataramani, Sidney Nagel, Robert Gerosh, and their students are studying crumpling. Take any old piece of paper, except perhaps the sheet you are reading now. Crumple it into a ball. Look at the ball. It has points and ridges on it. They emerge because the sheet can bend much more easily than it can stretch. When that is so, solid mechanics requires that the competition between bending and stretching energies naturally produces a high concentration of deformation energy near points and ridges, and thereby shares energy between these two kinds of deformations.

It is tempting to describe this concentration as an effect of a nonequilibrium situation. In equilibrium problems, the familiar equipartition law tells us that added thermal energy tends to be spread out equally among all modes. In contrast, in nonequilibrium problems, energy often tends to concentrate itself in a few modes or into a small region of space. A laser concentrates energy into one or a few modes of oscillation. Cold fusion was supposed to work by having energy concentrated into some nuclei. Apparently, that did not work, but in sonoluminescence, energy does concentrate itself so that incoming phonons, with energies in the nanovolt range, are converted into photons with energies of order of a volt or so. In a crumpled piece of paper, the vast majority of the energy that goes into bending the sample appears very close to the fold-lines. Once again, energy is wonderfully concentrated into a very small portion of the entire system. But, of course, your crumpled ball is just sitting there, in apparent equilibrium. Or maybe it is just stuck. Certainly one should ask, What kinds of physical systems do produce very strong concentrations of energy?

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