

# TURBULENT HEAT FLOW: STRUCTURES AND SCALING

**F**or very many years, scientists have studied the motion of enclosed fluids heated from below and cooled from above.<sup>1</sup> The containers for these Rayleigh–Bénard systems<sup>2</sup> have ranged in size from soda cans to swimming pools. Usually, as a fluid is heated it will become less dense. A heated blob will feel a force pushing it upward, the blob will tend to rise, and cooler fluid will fall into its place. At the lowest heating rates, there is no motion. Then, as the heating rate is increased, one sees successively a steady motion, a periodic oscillation, and a chaotic domain. At yet higher heating rates, one finds turbulent motion in which the fluid swirls in highly structured but never-repeating patterns (see figure 1).

Turbulence can be seen all around us. Waves on the sea and the swirling winds are proverbially inconstant. Their pattern changes and changes and changes again. Nonetheless, wind is in some sense always the same. Its statistical and average properties are predictable.

At least four strategies will isolate features of turbulent flows for scientific study. One is to look for the qualitative geometry of characteristic structures recurring in the flow. The second is to analyze fluctuations, looking for characteristic probability distributions. A third is to obtain quantitative characterizations of the average flows. Finally, one can measure and study the space and time dependence of velocities and other observables. In each case, we are trying to isolate elements of the flow that are open to prediction, replication, and comparison among different systems. I discuss the first three here, but put aside the last one because that subject is too large and is changing too rapidly.

## Structures: plumes, flywheels, and more

Figure 1 shows the temperature pattern in a turbulent Rayleigh–Bénard cell containing a rather viscous fluid. From studies of other fluid motion, one can expect this flow to contain reasonably well-defined structures, appearing in a mosaic of different combinations and ever-changing patterns.

One structure often found in heated fluids is called a plume. As heated fluid rises, it pushes aside the material above it and is, itself, in turn deflected. The rising material produces a stalk, while the deflected fluid produces a cap on top. As the pushing and deflection continue, the edge of the cap may further fold over. The result is something that looks like a mushroom. Figure 2 shows a very large plume produced by the rising gases of a nuclear explosion. Yet larger plumes are depicted in figure 3,

Geometrical structures and scaling behavior provide insights into the nature of convective turbulence and some risky generalizations about “complex systems.”

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carefully and uniformly from below, cooled from above, and illuminated with a nearly parallel light beam. Inhomogeneities in temperature bend the light and produce the bright lines and shadows shown. As you can see, the container is filled with plumes. Hot plumes congregate in an upwelling jet of fluid near the right-hand wall of the container. A similar, downward jet formed from cold plumes occurs on the left-hand wall. Large numbers of hot plumes are also found in left-to-right motion in a *mixing zone*, or viscous boundary layer, near the bottom of the container. A similar layer on the top contains cold plumes, moving from right to left. The central region contains a few plumes, hot and cold, in a partially random motion. These plumes wander chaotically, but also participate in an overall counterclockwise motion. In addition, there are very thin boundary layers, not really visible in the figure, near the top and bottom walls. The majority of the temperature drop between the bottom and top of the container occurs within these thermal boundary layers.

So, the heat flow has created a “machine” containing many different working parts: boundary layers, mixing zones, jets, and a central region. Figure 4 is a cartoon of the machinery at work. To follow the process, start at the lower left-hand corner of the convective cell. A wind produces flow from left to right, carrying with it some waves. Here, the waves are bulges in the height of the thin boundary layer at the bottom of the cell. As they move, the waves throw up a hot spray that tends to rise, forming sheets of hot, upward motion. These sheets break up into columnar structures that form the plumes shown in figures 1 and 4.

As they move across the bottom of the cell, these structures grow larger, as shown in figure 5. A few plumes come loose and move into the central region. Most hit the right-hand wall and move upward as a jet aimed toward the top of the cell. As a plume hits the upper wall, it makes a splash. The splash makes a wave. Each wave moves along the top, producing a cold spray and thence cold plumes. The plumes form into a downward jet at the left-hand side, splash on the bottom, and produce hot waves, thus keeping the motion going indefinitely. The jets pull the fluid around the container, forming a liquid flywheel. We have found an intricate motion that has the seeming purpose of moving heat from the bottom of the container to the top.

Notice that the depicted motion is counterclockwise.

which was taken from a computer simulation of the surface of the Sun. That picture shows many cold plumes falling downward into the Sun. More mundane plumes can be seen in a wide variety of laboratory pictures and simulations.

Now let us turn back to figure 1, which shows a small container heated

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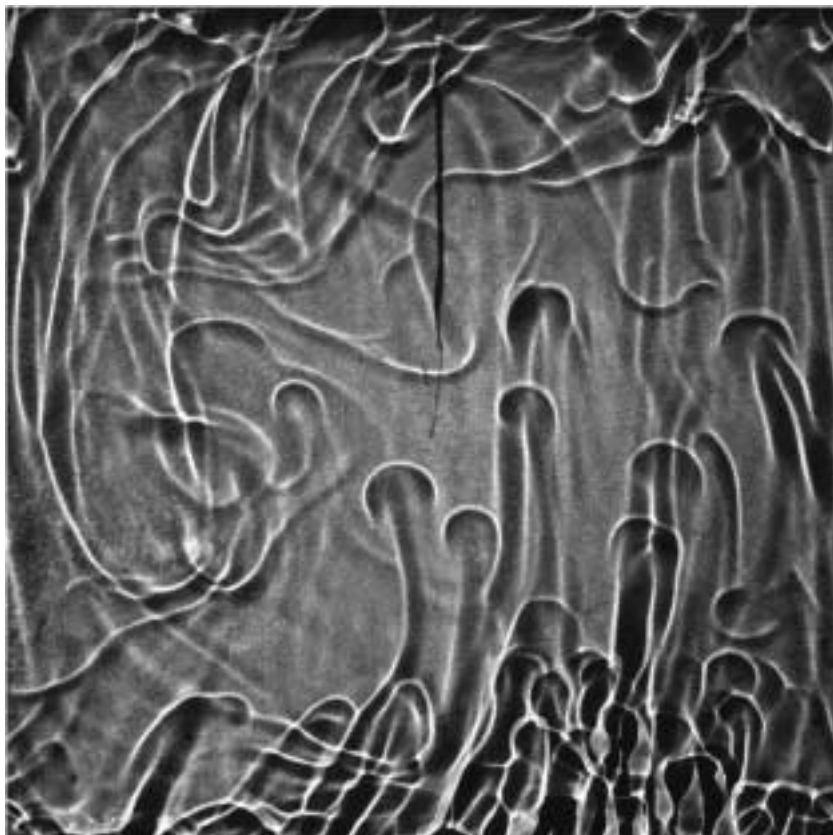


FIGURE 1. TURBULENT MOTION of glycerol in a container heated from below and cooled from above. The bright lines show regions of rapid temperature variation. The fluid contains many "plumes," especially near the walls. The plumes can be identified as mushroom-shaped objects with heat flowing through the "stalk" and spreading in the "cap." The hot plumes tend to rise with their caps on top; falling, cold plumes are cap-down. All this plume activity is carried along in an overall counterclockwise "wind." Note the thermometer coming down from the top of the cell. Figure adapted from J. Zhang, S. Childress, A. Libchaber, *Phys. Fluids* 9, 1034 (1997).

This direction of flow might have arisen from small inhomogeneities in the container when the system was first heated up. But clockwise motion is just as good as counterclockwise. Every once in a while the system will undergo a big change, in which the motion will reverse directions. In a perhaps analogous phenomenon, Earth's magnetic field reverses itself at apparently irregular intervals.

So, I have told a story. Let me point to a few lessons that can, if you wish, be extracted from that story.

The story is of a cell filled with a fluid. The fluid is strongly heated from below and cooled from above. Buoyancy raises the heated material and a flow starts.

The first lesson is that a nonequilibrium system can organize itself in an amazing fashion, leading to a sort of machine with many different parts, each serving an apparent function. This natural tendency toward self-organization might perhaps have some role in creating the intricate machinery of biological systems. Second, interesting and persistent structures can exist and persist in a noisy environment. Here, the structures include both the plumes and the counterclockwise flywheel motion of the entire fluid. If you observe the cell for a long time, you learn a third lesson: Big events happen. The occasional reversal of the flywheel is quite infrequent, but also quite dramatic. Many other complex things undergo sudden large changes, producing outcomes like tornadoes, or earthquakes, or rapid global warming.

The critical reader will recognize that the analysis up to this point has been rather subjective. I could have told another story, focusing on different features of the flow, and perhaps arrived at different lessons. Nonetheless, a multiplication of examples might help the reader believe that these words catch some general properties of complex systems. (The lessons might be more useful if one could give a precise definition of a "complex system." Unfortunately, there is no consensus on the definition. I like to think about situations in which there are many chaotically varying degrees of freedom interacting with one another.)



FIGURE 2. THERMONUCLEAR EXPLOSION "Joe 4," detonated at the Semipalatinsk Test Site, Kazakhstan, 12 August 1953. Plumes like this have served as the proverbial clouds "no bigger than a man's hand," (1 Kings 18:44) sitting on the horizon for everyone in the past 57 years.

## Basic equations—reliable knowledge

In contrast to the heuristic qualities of the lessons, we do have specific and exact knowledge of hydrodynamics. Scientists know the partial differential equations obeyed by any fluid in which variations in space and time are sufficiently gentle. These equations describe the flow of energy, momentum, and particles through the system. They relate rates of change in time to rates of change in space for velocity  $\mathbf{u}$ , temperature  $T$ , and pressure  $p$ . If the density  $\rho$  is almost constant, the full fluid equations can be simplified to the Boussinesq equations:<sup>3</sup>

$$\begin{aligned} \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} &= -(\nabla p) / \rho + \nu \nabla^2 \mathbf{u} + \alpha \mathbf{g} \\ \nabla \cdot \mathbf{u} &= 0 \\ \frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T &= \kappa \nabla^2 T. \end{aligned} \quad (1)$$

Here,  $\alpha$ ,  $\kappa$ , and  $\nu$  are, respectively, the thermal expansion coefficient, thermal diffusivity, and kinematic viscosity of the fluid, all assumed constant, while  $\mathbf{g}$  is the acceleration of gravity. Equations 1 have a very different epistemological status from the preceding “lessons.” The equations have been checked experimentally, studied theoretically, and used for many years in simulations.<sup>4</sup>

The Boussinesq equations are an essential and reliable part of our accumulated knowledge of fluids. Many observations and simulations confirm that plumes are one of their consequences. Indeed, figure 3 was generated in a computer simulation using a variant of equations 1.

Another very solid prediction can be made by converting the Boussinesq equations for the Rayleigh–Bénard cell into dimensionless form. The converted equations depend on two dimensionless parameters: the Rayleigh number  $Ra$ , characterizing the strength of the heat flow driving the turbulence, and the Prandtl number  $Pr$ , which measures the relative propensities of the fluid to diffuse momentum and heat. In symbols

$$Ra = \frac{g\alpha L^3}{\kappa\nu} \Delta T \quad \text{and} \quad Pr = \nu/\kappa, \quad (2)$$

where  $L$  is the depth of the container and  $\Delta T$  is the temperature difference from top to bottom. All containers of

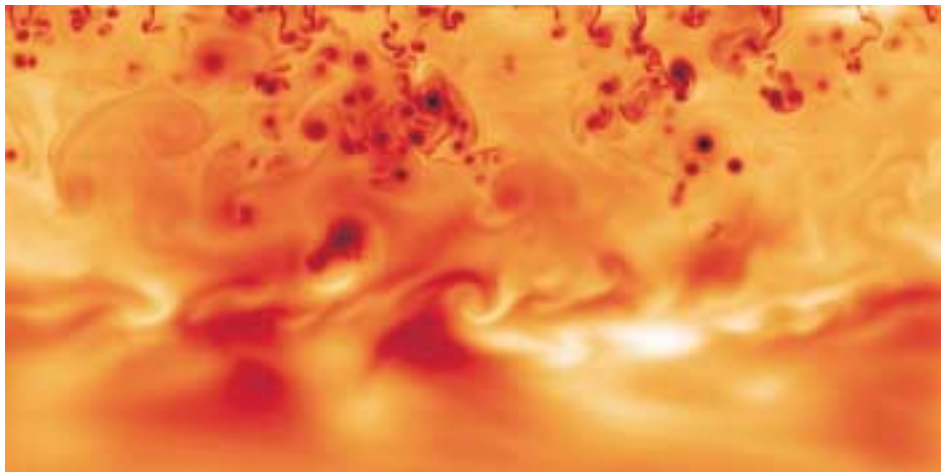


FIGURE 3. COMPUTER-SIMULATED TEMPERATURE pattern near the surface of the Sun. The surface is at the top of the picture. The darker colors indicate lower temperatures. Notice the many falling plumes near the surface. The simulation is by Andrea Malagoli, Anshu Dubey, and Fausto Cattaneo, all at the University of Chicago.

the same shape and obeying the same boundary conditions will have their flow properties completely determined by the Rayleigh and Prandtl numbers. (For example, the fluid in figure 1 has a Rayleigh number of  $2.3 \times 10^8$  and a Prandtl number on the order of hundreds.)

## Exponential tails and spectacular events

Large excursions are a regular feature of the behavior of turbulent systems. We have already discussed turnabout in the flywheel motion of our Rayleigh–Bénard example. In addition, the temperature in the central region shows an occasional very large excursion, probably as the result of the passage of fluid newly liberated from far away.<sup>5</sup>

To sharpen the notion of “occasional large excursion,” one might consider the probability distribution for observing a given temperature as a function of the deviation of that temperature from its average value. Since the time of Francis Galton and Johann Gauss, those studying statistical systems have assumed that, under most circumstances, the probability distribution for a deviation of size  $\Delta$  will have a Gaussian form. That is, the distribution will be proportional to an exponential of a positive constant times  $-\Delta^2$ . However, turbulent flows produce an enhanced likelihood for large fluctuations, as manifested by a probability distribution that falls off much less steeply than a Gaussian. Specifically, the observed<sup>6</sup> temperature distribution is proportional to an exponential of a positive constant times  $-|\Delta|^p$ , with  $p$  between 1 and 2. Just below the onset of strong turbulence,  $p$  is 2—the temperature distribution is Gaussian. As shown in figure 6, in the lower part of the strongly turbulent range,  $p$  is close to 1.0. For a higher range of Rayleigh numbers,  $p$  is about 1.5.

A  $p = 1$  distribution for extreme events, called an exponential tail, appears to be a characteristic feature of many highly disordered physical situations. Exponentials are seen, for example, in force distributions in glassy materials, foams, and sand piles.<sup>7</sup> We do not understand why they appear so often.

Smaller  $p$  means that big fluctuations are more likely. The increased likelihood of extreme events can lead to extreme consequences. If, for example, you were an engineer designing an airplane part with a potential for causing in-flight failure, you might consider it to be conservative if you designed for safety against all events within a 10 standard-deviation range. Indeed, with a Gaussian distribution, bad events would have a probability of about 1 in  $10^{22}$  of occurring—almost certainly good enough. With an exponential distribution, unfortunately, the catastrophic event has a likelihood of 1 chance in 20 000. A big difference.

In addition to giving us information about the likelihood of large temperature excursions, figure 6 also shows the accuracy and robustness of the experimental measurements possible in this field. Events with deviations from the mean of eight  $\sigma$  are seen and measured. Exponential behavior, which would be represented by a fit to the data with a pair of straight lines, is a good approximation to the data—but not perfect. Curva-

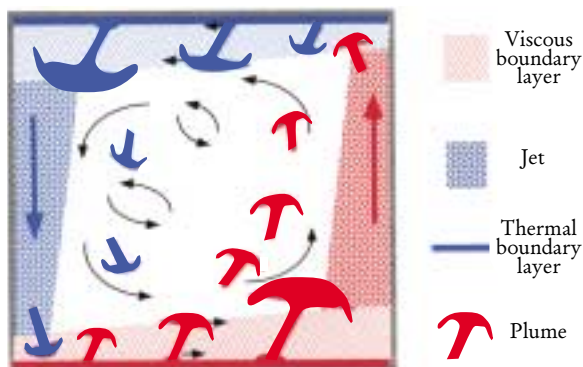


FIGURE 4. THE RAYLEIGH-BÉNARD CELL as a machine, schematically drawn. The red-shaded areas of the cell show regions with hot fluid, while the blue areas indicate cold fluid. The arrows give the direction of fluid flow. Compare this illustration with the “real thing” shown in figure 1. A major point of the cartoon is to list and show the many different structures that work together to make the intricate motion of the convective system. As described in the text, the plumes play a very important role both as motors and as heat pumps. The illustration is partially based on a cartoon in G. Zocchi, E. Moses, A. Libchaber, *Physica A* 166, 397 (1990). See also the description of the motion in X.-L. Qiu, P. Tong, *Phys Rev. E* (in press).

ture is discernible in both peak and wings. The whole behavior, curvature and all, is seen to be experimentally reproducible in that it is very close to the shape seen by other experimentalists and is also the same at all three Rayleigh numbers. This independence from Rayleigh number is an important hint for theorists, and suggests that they would wish their eventual theory to give “universal” results, that is, outcomes with substantial robustness against changes in the Rayleigh number.

### Algebraic characterizations

The total amount of heat flowing upward through a convective cell is measured by the Nusselt number  $Nu$ , which is the ratio of the actual heat flow to the flow that would occur via heat conduction alone. The motion of the fluid enables  $Nu$  to become increasingly large as the Rayleigh number increases (see figure 7).

Many theories begin from a picture of the different regions within a convective cell and then estimate the orders of magnitude of the temperature and various velocities in each region. In such theories, it is traditional to guess that there is a simple scaling relation between the dimensionless quantities that parameterize the system. For the Rayleigh–Nusselt relation, this guess is reflected in a power law of the form

$$Nu = A Ra^\beta + \dots, \quad (3a)$$

where the ellipses represent corrections to the simple power law. I return to the idea of corrections to simple descriptions in a moment.

In one such theory, a University of Chicago collaboration (see the second entry in ref. 6) identified the different regions shown in figure 4 and then estimated typical temperature differences, velocities, and region sizes to obtain a power law with  $\beta = 2/7$ , fitting the then-existing data and some subsequent data pretty well.<sup>8</sup>

Other “laws” do the same job, and perhaps even better. For example, in a recent paper,<sup>9</sup> Xiaochao Xu, Kapil M. S. Bajaj, and Guenter Ahlers of the University of Cali-

fornia, Santa Barbara, got an excellent fit using the suggestion of Siegfried Grossmann and Detlef Lohse<sup>1</sup>:

$$Nu = A Ra^{1/3} + B Ra^{1/4} + \dots \quad (3b)$$

To obtain equation 3a, the Chicago group assumed a more or less serial process of heat movement from boundary layer to mixing zone to central region, while the use of equation 3b assumes a pair of processes working in parallel to produce two terms.

Other fits are possible. Joseph Niemela of the University of Oregon, and collaborators<sup>10</sup> note an excellent fit,

$$Nu = A [Ra^{3/2} (\ln Ra)]^{1/5} + \dots, \quad (3c)$$

from a theory based on the flow through a single “optimum” mode. They also point to an equally good fit via the simple power law (equation 3a) with  $\beta = 0.309$ . All these proposed formulas match the data to a respectable accuracy (see figure 7). How to choose?

Before choosing it is reasonable to ask, “What are we trying to do?” If our goal is to get a decent representation of the facts observed in a very wide range of turbulence experiments, all three formulas are acceptably good.

A different perspective might be in order. None of these equations means anything in themselves. To give them meaning, you have to define the terms “+...” appearing in all three equations. In my view, the right thing is to demand that the fit become *asymptotically accurate*. I mean that there should be some limiting process in which the proposed theory would be exactly true.<sup>11</sup> The limiting process would most likely involve having the Rayleigh number go to infinity, with maybe also having the Prandtl number going to some extreme value. Alternatively, some fluctuations or processes might be neglected. Then the “+...” would represent the corrections to this asymptotic result and could be given a well-defined meaning. So, I am suggesting that, to fit the data, one should look forward and guess the form of the eventual mathematical theory that will describe what happens in the system in some very extreme limit.<sup>12</sup>

One possible description of such an eventual theory might be discerned in Robert Kraichnan’s work of 1962, which outlines three different asymptotic regimes.<sup>13</sup> One regime is very, very high Rayleigh number with a nonextreme value of

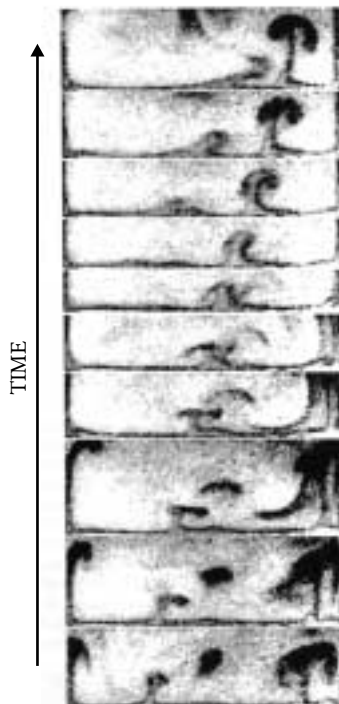


FIGURE 5. PLUMES MOVE across a bottom boundary layer and hit a side wall. Time goes upward in this series of experimental visualizations. Here, there is an overall left-to-right “wind” along the bottom of the tank. Notice how the plumes form on top of the waves, ride with the wind, and are squeezed into the side wall. Figure adapted from B. Gluckman, H. Willaime, J. Gollub, *Phys. Fluids A* 5, 647 (1993).

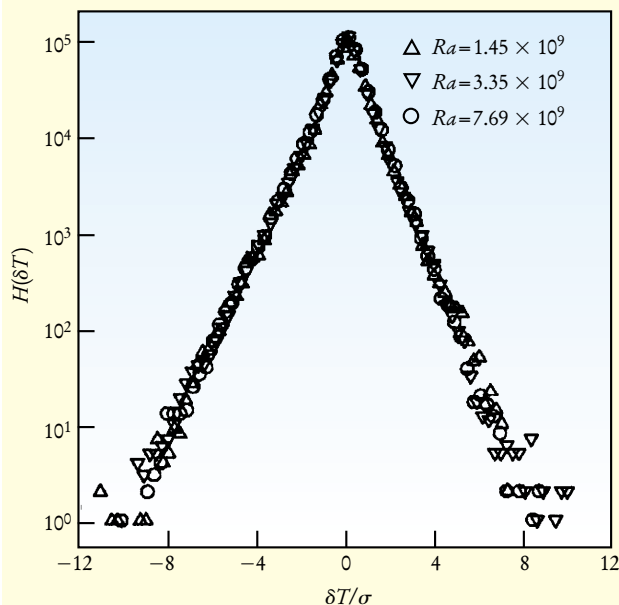


FIGURE 6. FLUCTUATION PROBABILITY for temperature near the center of a convective cell, in the lower part of the strongly turbulent range of Rayleigh number (adapted from Du and Tong in ref. 6). For each value of  $Ra$ , the temperature data are normalized so as to keep their standard deviation fixed. The fluctuations graphed are considerably larger than would be expected from a Gaussian distribution. The anomalously large fluctuations are probably produced by plumes that can carry temperature fluctuations from place to place over long distances.

the Prandtl number. In that case, Kraichnan said, highly energetic turbulent events would destroy much of the structure shown in figure 1. Globbs of hot fluid would move from bottom to top in an almost ballistic fashion, trading their  $g\alpha\Delta T$  potential energy for kinetic energy, thereby generating a behavior described by equation 3a with  $\beta = 1/2$  (perhaps modified by logarithmic corrections). However, Kraichnan and, more recently, Grossmann and Lohse<sup>1</sup> have argued that this “ultimate” regime requires tremendously high Rayleigh numbers. Consequently, almost all actual experiments should be understood as being in regimes different from the “ultimate” one. The other regimes have more structure in the cell.

Another behavior discussed by Kraichnan might be seen at a rather large Rayleigh number (in the range from about  $10^8$  to  $10^{24}$ ) and high Prandtl number. Many experiments for convection in water and helium fall into this region. Kraichnan also suggests a third regime, of relatively high Rayleigh but low Prandtl number, that might describe convection in mercury.<sup>14</sup> According to Roberto Verzicco and Roberto Camussi (see second entry of ref. 4), the difference between the last two regimes is whether the main heat flow is carried by plumes or by the “flywheel.”

Indirect experimental evidence tends to support the notion of several asymptotic regimes: Different experimental groups can observe different apparent values of the index  $\beta$  even when they are working with the same fluid and the same value of  $Ra$ , but have cells that differ in geometry or in boundary behavior. Reference 14 refers to papers that show such a “discrepancy” for the low Prandtl number fluid, mercury. In their studies of helium,<sup>15</sup> Xavier Chavanne of the Ecole Normale Supérieure

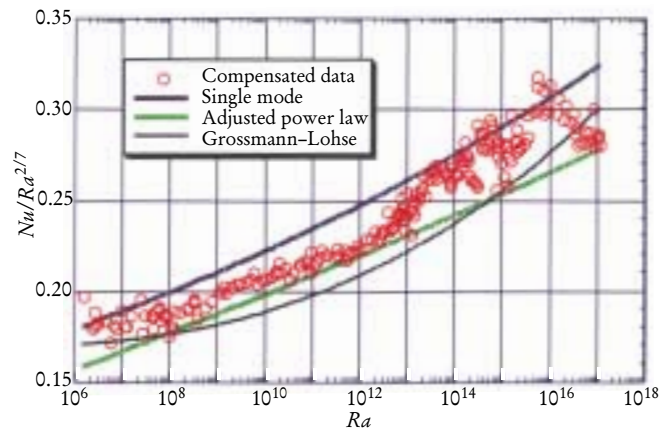


FIGURE 7. RA-NU RELATIONS. The graph shows the dimensionless heat flow  $Nu$  plotted against the dimensionless temperature difference  $Ra$ . In each case, the  $Nu$  values are compensated by being divided by  $(Ra)^{2/7}$ . The experimental data are shown by red circles and are derived from Niemela et al., ref. 6. The power law has  $\beta = 0.309$  as suggested by Niemela et al. The other two curves depict theoretical heat-flow calculations and are of the forms indicated in equations 3b and 3c. In each case, the fitting curves have been rigidly displaced from their best (least squares) positions to better show the data. Not one of the theoretical calculations makes use of the plumes so evident in several of the previous figures. This lack suggests that we do not yet fully understand what we are seeing in the Rayleigh-Bénard system.

in Paris and his collaborators observed a  $\beta = 1/2$  regime in exactly the same range of  $Ra$  in which Niemela and colleagues<sup>6</sup> found  $\beta = 0.309$ . Such differences would be understandable if a difference in boundary conditions could tip convective behavior into different asymptotic regimes.<sup>16</sup>

The theories that characterize the different asymptotic regimes all include some scaling characterizations of the various regions and structures within the flow. Thus, detailed measurements of temperatures and velocities as a function of position within the cell can serve to distinguish among the possible theories and regimes.<sup>17</sup> Unfortunately, these measurements are very difficult to perform at the highest values of the Rayleigh number because structures in the convection cell can be very small—with length scales down into the micron region.

## Lessons from complexity

In recent years, there has been much discussion of the nature of complexity in physical systems.<sup>18</sup> At one time, many people believed that the study of complexity could give rise to a new science. In this science, as in others, there would have been general laws, with specific situations being understandable as the inevitable working out of these laws of nature. Up to now, we have not found any such laws. Instead, studies of specific complex situations, for example the Rayleigh-Bénard cell, have taught us lessons—homilies—about the behavior of systems with many independently varying degrees of freedom. These general ideas have broad applicability, but their use requires care and good judgment. Our experiences with complex systems encourage us to expect richly structured behavior, with abrupt changes in space and time, and some scaling properties. We have found quite a bit of self-organization and have learned to watch out for surprises and big events. So, even though there is apparently no science of complexity, there is much

science to be learned from studying complex systems. Finally, we have learned that no theory of everything can include every interesting thing.

*I have had helpful discussions with Guenter Ahlers, Emily Ching, Russell Donnelly, Detlef Lohse, Elisha Moses, Joseph Niemela, Boris Shraiman, Katepalli Sreenivasan, Victor Steinberg, Penger Tong, and Jun Zhang. This work was supported by the NSF via the MRSEC program and a series of DMR grants, most recently NSF: DMR-0094569.*

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