



# A precursor superconductivity approach to magnetic field effects in the pseudogap phase

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## Abstract

We study the upper critical field  $H_{c2}$  (or  $T_c(H)$ ), and  $T^*(H)$  within the pseudogap state, using a mean field scheme which incorporates a strong pairing attraction; for small coupling,  $g$ , this approach reduces to BCS theory. However at larger  $g$  a pseudogap,  $\Delta_{pg}$  is present at  $T_c$  which dramatically changes the behavior of the inferred coherence length  $\xi$ , relative to that of BCS theory. Similarly, the quadratic terms in a Ginzburg–Landau description, are modified by  $\Delta_{pg} \neq 0$ . These pseudogap effects, in accord with experiment, are responsible for the fact that (i)  $\xi$  is weakly  $x$ -dependent (ii) and  $T_c$  is more strongly  $H$  dependent than is  $T^*$ . Very near the superconductor–insulator transition  $\xi$  is predicted to quickly increase. © 2002 Elsevier Science B.V. All rights reserved.

*Keywords:* Critical fields; Pseudogap; Precursor superconductivity

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The various approaches to understanding the pseudogap phase of the high temperature superconductors seem to be divided roughly into two schools: those in which the pseudogap is associated with a competing energy gap (or hidden order parameter [1]) and those in which the pseudogap derives from the superconductivity itself. This latter “precursor superconductivity” school has multiple interpretations as well. The phase fluctuation [3] or the spin-charge separation schools [2] are to be distinguished from the present scheme in which the pseudogap is associated with a strong(er than BCS) pairing attraction. In the presence of the latter, pairs form at a higher temperature than that at which they Bose condense. This is often referred to as a BCS–Bose Einstein crossover scenario.

In the present paper we apply a formalism which we have extensively discussed and developed previously [4–7] to understand magnetic field effects in the pseudogap phase. Our approach is to be differentiated from the phase fluctuation school which builds on strict BCS theory (a special limit of the more general mean field theory we consider), but adds fluctuation effects which we neglect here. Of fundamental importance here

is the fact that as the coupling strength  $g$  increases, there are both fermionic and bosonic excitations; below  $T_c$  the latter have a dispersion relation of the BCS form with excitation gap  $\Delta$  given by  $\Delta^2 = \Delta_{sc}^2 + \Delta_{pg}^2$  which is to be distinguished from the order parameter  $\Delta_{sc}$ , at all non-zero temperatures. Here  $\Delta_{pg}$  corresponds to the pseudogap which results from the presence of finite momentum (bosonic-like) pair excitations.

It is the presence of this excitation gap at  $T_c$  which differentiates the physics of the underdoped cuprates from that of the overdoped (BCS) state. We here associate over- and under-doping with small and large normalized coupling constants. In all our calculations  $g$  enters in a dimensionless form as a ratio to the bandwidth. It is presumed that as the Mott insulator is approached, the characteristic electronic energy scales decrease—so that even if  $g$  is relatively  $x$ -independent, its effectiveness increases with underdoping. There is, in addition, another important effect of variable  $g$ . As this quantity increases the pair stiffness decreases. These observations may be incorporated into a Landau–Ginzburg free energy functional, with quadratic terms only, given by

$$F = \left( \bar{\tau}_0 \left( 1 - \frac{T}{T_c} \right) + \eta^2 \left( -i\nabla - \frac{2e\mathbf{A}}{c} \right)^2 \right) |\Delta_{sc}|^2. \quad (1)$$

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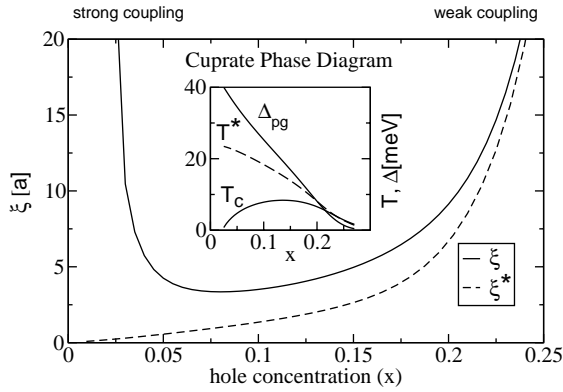


Fig. 1. Magnetic length scales associated with  $T_c$  and  $T^*$  as a function of hole concentration in the cuprates. Zero field results from Ref. [6] are shown in the inset ( $\Delta_{pg}$  is a factor of 2 smaller than experiment due to our convention for  $\varphi_k$ ).

Here  $\bar{\tau}_0$  represents density of states effects, and  $\eta$  the stiffness parameter. Both of these are in principle dependent on the strength of the coupling  $g$ , directly as well as indirectly through the introduction of the pseudogap. On this basis we define the coherence length  $\xi$  as

$$\frac{1}{T_c} \frac{dT_c}{dH} = \frac{2\pi\eta^2}{\Phi_0 \bar{\tau}_0} = \frac{2\pi}{\Phi_0} \xi^2. \quad (2)$$

This coherence length contains two competing effects via pair stiffness and density of states, and as a result we find that  $\xi$  is roughly independent of the strength of  $g$ , except at the extreme strong coupling limit. Translated into the variable  $x$ , we find as in experiment, that  $\xi$  is roughly  $x$ -independent, except very close to the superconductor-insulator boundary where it increases dramatically, which reflects the ideal “boson” limit, where  $T_c$  is suppressed [8] to zero at any  $H \neq 0$ .

The low  $H$  calculational framework for computing  $\xi$  and its counterpart for  $T^*$  called  $\xi^*$  is based on the semi-classical approximation, combined with a calculation of the zero field pair susceptibility [9]. The details are given

elsewhere [10]. Fig. 1 shows the behavior of  $\xi$  and  $\xi^*$  vs. hole doping concentration. Over most of the range of  $x$ ,  $\xi$  is relatively constant, as seems to be observed experimentally, and its magnitude is within a factor of two or three of experiment [11]. As in experiment,  $T^*$  is found to be less field sensitive in the underdoped [12] than overdoped [13] regimes. As the insulator is approached,  $\xi$  rapidly increases [2] while  $\xi^*$  continues to decrease. We have thus demonstrated that the different observed field dependences of  $T^*$  and  $T_c$  (for both under- and overdoped cuprates) are contained in our theory, in which the pseudogap is associated with precursor superconductivity.

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