Bridging by Light

Robert Schroll
Wendy Zhang

University of Chicago
Can we explain the stability of these shapes?
Can we explain the sizes of these shapes?
Forces on Surface

- **Surface tension:**
  \[ P_\sigma = 2\sigma H \]
  \[ H = \frac{1}{2} \left[ \frac{r''(z)}{(1 + r'(z)^2)^{3/2}} - \frac{1}{r(z) (1 + r'(z)^2)^{1/2}} \right] \]

- **Gravity:** \[ P_g = \Delta \rho g z \]
- **Radiation pressure:** \( P_{rad} \)
Radiation Pressure

- Radiation pressure comes from momentum discontinuity at surface
- Momentum of photon:
  \[ p_\gamma = E_\gamma n/c \]
- For a single photon:
  \[ \Delta p = \frac{E_\gamma}{c} n_2 \cos \theta_2 \]
  \[
  \cdot \left[ 1 + R - \frac{\tan \theta_2}{\tan \theta_1} T \right]
  \]
  \[
  T = \frac{1}{2} \left( \frac{4 n_1 n_2 \cos \theta_1 \theta_2}{(n_2 \cos \theta_1 + n_1 \cos \theta_2)^2} + \frac{4 n_1 n_2 \cos \theta_1 \theta_2}{(n_1 \cos \theta_1 + n_2 \cos \theta_2)^2} \right), \quad R = 1 - T
  \]
Illumination of surface by intensity $I(r)$:

$$P_{rad} = \frac{n^2}{c} \cos^2 \theta_2 \left[ 1 + R - \frac{\tan \theta_2}{\tan \theta_1} T \right] I(r)$$
Previous work has explained low-power dimples as a static balance of these three forces.

Casner and Delville, Phys. Rev. Lett. 87, 054503
Under the influence of only surface tension, a cylinder will be unstable if its length is greater than its circumference.

At long lengths, breaking into a series of bubbles is energetically more favorable.
Radiation pressure must be holding bridge open

Assume a flux density of photons $\Phi_0$ trapped by TIR (total internal reflection)

Gives pressure $P_{rad} = \frac{p_\gamma}{2} \frac{\cos \phi}{\tan \phi} \Phi_0$

If radius varies,

$$P_{rad} = \frac{p_\gamma}{2} \frac{\cos \phi}{\tan \phi} \Phi(z) = \frac{p_\gamma}{2} \frac{\cos \phi}{\tan \phi} r(z)^2 \Phi_0$$
Energy Analysis:

- Change in surface energy moving from straight to sinusoidal walls, preserving volume
- Work done against radiation pressure to reach that shape
- Deformed surface always costs more energy, so varicose instability is overcome.
- Pressure analysis is more clear (in long wavelength case)
Bridge Stability

- Assume a static solution exists:
  \[ P_0 = \frac{\sigma}{r_0} + \Delta \rho g z \]

- Create a disturbance of characteristic sizes \( \Delta r, \Delta z \)
  \[ (\frac{\Delta r}{\Delta z} \ll 1) \]

- Calculate lowest order change in pressure
Bridge Stability

\[ \Delta P = \sigma \Delta r \left( -\frac{1}{\Delta z^2} + \frac{1}{r_0^2} \right) + 0 - 2 \frac{\Delta r}{r_0} P_0 \]

\[ = -\Delta r \left( \frac{2P_0}{r_0} + \frac{\sigma}{\Delta z^2} - \frac{\sigma}{r_0^2} \right) \]

Static solution means: \[ P_0 = \frac{\sigma}{r_0} + \Delta \rho g z > \frac{\sigma}{r_0} \]

\[ \Rightarrow \frac{2P_0}{r_0} + \frac{\sigma}{\Delta z^2} - \frac{\sigma}{r_0^2} > \frac{\sigma}{r_0^2} + \frac{\sigma}{\Delta z^2} > 0 \]

Thus, \( \Delta P \) and \( \Delta r \) will always have opposite signs. The change in pressure will oppose the change in \( r \).
Bridge Stability

- Bends will be unstable:
  - The side bending in will feel a large increase in radiation pressure
  - The side bending out will experience a decrease in radiation pressure
Bridge Formation

- Bridge is always stable
- Difference between dimple and jet is energy to deform surface
- Explains hysteresis
- Jet becomes bridge when it hits the bottom
  - Upper fluid wets container
- Why does bridge form and disappear at the same power?