3-1 book ch. 2 Law of Averages for $\langle r^2 \rangle$

A polymer in one dimension consists of $n$ vectors $\vec{a}_1, \vec{a}_2, \ldots, \vec{a}_n$ of length 1 that point randomly left or right. Each vector is independent of the others. a) Find $\langle \vec{a}_5 \rangle$, $\langle \vec{a}_5 \cdot \vec{a}_5 \rangle$ and $\langle \vec{a}_4 \cdot \vec{a}_5 \rangle$. b) Using averages such as these, and noting that $\vec{r} = \sum_i \vec{a}_i$, find $\langle \vec{r} \rangle$ and $\langle \vec{r}^2 \rangle$ as a function of $n$.

3-2 Short lattice chains

A four-step polymer in one dimension consists of left- and right-directed bonds of unit length. a) How many configurations of the chain have the end four units to the right of the beginning? How many have the end three steps to the right? Two? one? zero? You may find it easiest to draw a picture of all $2^4$ configurations. b) Find $\langle r^4 \rangle / \langle r^2 \rangle^2$ for this polymer and compare with the corresponding ratio for a one-dimensional Gaussian chain, i.e., 3.0.

3-3 Toy Renormalization, based on book 3.3

In the text we deduced a way of inferring polymer behavior by analyzing the scaling properties of the equation without solving it. This problem is meant to show the same procedures in the familiar context of classical mechanics. Here we wish to find the limiting properties of a system’s motion as the energy of a particle goes to zero. The potential energy function $U(x)$ of a point particle of mass $m$ has the form $U(x) = U_0[1 - \cos(|x|^{1/6})]$. A particle with energy $E \ll U_0$ is trapped in the region near $x = 0$. It is obliged to oscillate with some period $\tau$. The object is to find how the period $\tau$ varies as the energy $E \to 0$. a) Write conservation of energy for the particle in the region $x \ll 1$. Notice that in this region $U(x) \approx U_0 \left(\frac{1}{2}|x|^{1/3} - \frac{1}{6}|x|^{2/3}\right)$. In order to find the limiting behavior as $E \to 0$, define $\tilde{E} = \lambda E$ so that $\tilde{E}$ will stay finite when $E \to 0$ when expressed in terms of the $\tilde{\cdot}$ variables. We wish to find a corresponding time scale $\tilde{t} = \mu t$ and distance scale $\tilde{x} = \eta x$ such that the equation of motion will stay finite when $\lambda \to \infty$. b) Restate the conservation of energy from a) in terms of the $\tilde{\cdot}$ variables. c) How must $\mu$ and $\eta$ vary with $\lambda$ in order for the terms in the equation to stay finite? d) How does the temporal period $\tau$ vary with the maximum displacement in $x$ as $E \to 0$? e) If $\mu$ and $\eta$ vary with $\lambda$ as found in b), does the correction in $|x|^{2/3}$ become small or become large?

3-4 Series and Parallel Polymers

Two identical random-walk polymers are fastened together end to end, to make a chain of double the length of each. a) What is the root-mean-squared end-to-end distance $\sqrt{\langle r^2 \rangle}$ of the two free ends relative to that of a single polymer? b) The same two polymers are fastened together at both ends to make a two-chain polymer. What is the root-mean-square end-to-end distance between the two ends in this parallel configuration? *Hint*: What is the probability that two independent walks both reach the point $\vec{r}$ in $n$ steps?
3-5 Coil-Stretch Transition, based on book 3.5

An elongational flow field may be made by aiming two round nozzles at each other underwater, then sucking water out through both nozzles at the same rate. Any polymers at the center of this contraption are pulled towards both nozzles at once. The effect is roughly equivalent to a force pulling on each end, of magnitude \(bz^2\), where \(z\) is the distance from the center along the nozzle axis. The coefficient \(b\) is proportional to the elongation rate \(\dot{\gamma}\). One can detect the amount of elongation \(\langle z \rangle\) induced in the polymers by the flow.

**a)** What energy \(W(z)\) would a free particle starting at the origin gain if it moved to \(z\) under the action of this force? This energy is part of the energy of any polymer ending at \(z\) in the presence of the flow.

**b)** Denote the \(\langle z^2 \rangle\) of the polymer without flow as \(Z_2\). Treating the polymer as a spring, as explained in Chapter 3, sketch the work \(U(z)\) required to elongate the chain to length \(z\) as a function of \(z\) for small \(b\), counting both the elastic free energy of the polymer and energy in a), and write its functional form.

**c)** This energy \(U(z)\) attains a maximum for some elongation \(z^*\) and energy \(U^*\) (relative to its \(z = 0\) value). How does the height of this maximum vary with \(b\), when \(b\) is small? If the height \(U^*\) is adjusted to be the thermal energy \(T\), what is the corresponding position \(z^*\) of this maximum in terms of \(Z_2\)? (We have seen that this maximum amounts to an activation barrier. When it is higher than about \(T\), passage over the barrier becomes very slow.) According to this \(U(z)\), once \(z\) exceeds \(z^*\), it will increase to infinity. In reality \(z\) increases only to the maximum extension \(r_{\text{max}}\) of the polymer. This maximum extension is not accounted for in the Gaussian \(p(n, r)\), as the text points out.