P5.1 Thermocouple  The most obvious experiment to see the thermopower of a metal is to attach the ends of two wires made of different metals and then subject the two ends to the same temperature difference as shown in Fig 13.1. If one measures the open-circuit potential across in this current loop, one gets a voltage proportional to the temperature difference. This voltage is also proportional to the difference in thermopowers of the two metals. This effect provides a means of measuring temperature differences. Such a device is called a thermocouple.

a) Using methods of Chapter 2, predict the voltage per degree of temperature difference for two ideal Sommerfeld metals with Fermi energies $E_1$ and $E_2$.

b) What pair of metals in Table 2.1 should give the biggest voltage, and how big is it for a one-degree temperature difference?

c) The Handbook of Chemistry and Physics reports thermocouple coefficients for copper and constantin wires and for iron and constantin wires. Deduce the difference in thermopower of iron and copper and compare with your predictions using Table 2.1.

P5.2 reflecting and transmitting bandstructure  Cf. Problem 8.1. A one-dimensional crystal consists of a row of equally spaced atoms. These interact with electrons to form a periodic potential like that shown in Figure 8.4. Considering a single atom, an electron with energy $(\hbar K)^2/2m$ incident from the left will scatter from this potential. The transmitted wave has amplitude $t = |t| e^{i\delta}$, and the reflected wave has amplitude $r = \pm i\sqrt{1 - |t|^2} e^{i\delta}$. The same $r$ and $t$ hold for waves incident from the right. $r$ and $t$ depend on the incident energy in general. (The book suggests how to prove that $r$ and $t$ are related this way.) If the spacing between these potentials is $a$, there is a definite relation between the Bloch wavevector $k$ and the “free wavevector” $K$ defined above.

a) Determine $\cos ka$ as an explicit function of $K$, $|t|$ and $\delta$.


a) If there are two electrons per atom, and the interaction strength is very small compared to the Fermi energy, what fraction of the occupied states lie in the second-lowest band?

b) Suppose the smallest reciprocal lattice vector has magnitude $K_1$, and denote the lattice potential at these wavevectors by $U_1$. Suppose the unperturbed energy for $k$ at a corner of the Brillouin Zone is $\mathcal{E}_0$. Find the three energy lowest energy states for this $k$ in terms of $U_1$ and $\mathcal{E}_0$. 