P7.1 Anisotropic mass  (Cf. Problem 12.1) As we have seen, the maxima and minima of an energy band need not be isotropic. Thus the general form of the energy near a minimum is
\[
\varepsilon(k) = \text{constant} + \frac{\hbar^2}{2}(k - k_0) \cdot M^{-1}(k - k_0),
\]
where \(M\) is a 3×3 (symmetric) matrix called the mass matrix, and \(k_0\) is the position of the minimum. If \(k_0\) is an energy maximum, the second term is negative. For some metals and most semiconductors all the occupied or empty states are close enough to maxima or minima that \(\varepsilon(k)\) is well described by this mass-matrix formula.

a) For such a material find the density of states \(g(\varepsilon)\) and find the specific heat in terms of \(M\). That is, find the specific heat effective mass \(m^*\) of p. 48 in terms of \(M\).

b) For such materials the crystal momentum \(k\) is proportional to the velocity \(v\) in the sense \(v = \hat{A}k\) for some matrix \(\hat{A}\). Using Hamilton’s equations (i.e., the semiclassical formalism of Chapter 12), find the proportionality matrix \(\hat{A}\) in terms of \(M\).

c) What is the equation of motion giving the acceleration \(d\vec{v}/dt\) for such an electron in a uniform electric field \(E\) and magnetic field \(H\)?

d) Using the reasoning of chapter 1, wherein the steady-state average velocity \(\langle v \rangle\) is given by \(v(t)|_{t=\tau}\), find the generalization of the Drude formula \(\sigma = ne^2\tau/m\). That is, what expression involving \(M\) should replace \(m\) in the Drude formula? It is easiest to find the components of \(\sigma\) along the (orthogonal) principal axes of \(M\).

e) Find the cyclotron frequency \(\omega_c\) for electrons in a uniform field \(H\) an arbitrary axis. That is, what should replace \(m\) in the formula \(\omega_c = eH/(2\pi mc)\)? Part c) is a linear equation of motion, so that \(v\) must vary sinusoidally in time. \(\omega_c\) must then be the sinusoidal frequency.

P7.2 Magnetoresistance  Formula 13.69 and 13.70 tell how to find the conductivity in the presence of a static magnetic field \(H\) in the \(z\) direction. Suppose a density \(n\) of electrons has an \(\varepsilon(k)\) described by the mass-matrix formula above, with principal axes along \(x\), \(y\), and \(z\).

a) Use these formulas to find \(\sigma_{xx}\) and \(\sigma_{zz}\) to lowest nonvanishing order in \(H\) (i.e., \(O(H^2)\)) in terms of the mass matrix elements \(M_{xx}, M_{yy}\), and \(M_{zz}\).