Optimal Quantum Cloning on a Beam Splitter

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We demonstrate how a beam splitter in combination with different light sources can be used as an optimal universal $1 \rightarrow 2$ quantum cloner and as an optimal universal quantum NOT machine for the polarization qubit of a single photon. For the cloning a source of single photons with maximally mixed polarization is required and for the NOT operation a source of maximally entangled photon pairs. We demonstrate both operations with near optimal fidelity. Our scheme can be generalized in a natural way to clone and NOT the spin state of electrons.

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No device can be constructed that produces copies of a quantum system in an unknown quantum state. This feature of the linearity of quantum dynamics, known as the "no cloning" theorem [1], lies at the heart of one of the main differences between classical and quantum states: the state of a single quantum system cannot be ascertained by measurement; if one could measure it, one could copy it, and if one could copy it, one could measure it. On a practical level this theorem guarantees the security of quantum cryptography.

Given that a perfect quantum copying machine cannot exist, it is natural to ask how good an approximate copy-making machine can be constructed [2]. No copying machine can be made that both keeps the original state intact and outputs copies that contain any information about the original state. Consequently, the production of copies that bear some resemblance to the original will have to be accompanied by a distortion of the original state. Theoretical work has been dedicated to finding the best allowed transformation that gives an equally faithful output whatever the input. The answer is a transformation that involves no loss of information about the original state [2–5]. For the case of $1 \rightarrow 2$ copying of the state of a two-level system, the optimal transformation returns with a probability of $2/3$ two copies of the original state and with a probability of $1/3$ a copy of the original and a copy of the orthogonal complement of the original, in such a way that it is impossible to tell which is which.

Another result that highlights the differences between classical and quantum two-level systems is that no device can be constructed that maps an unknown quantum state of a two-level system onto its orthogonal complement. Such a transformation would be equivalent to a quantum version of the NOT gate; it is antiunitary since it maps every point of the Poincaré sphere onto its antipode, and thus cannot be constructed out of unitary evolutions [6].

The question of how good an approximate NOT can be implemented has also been addressed [6] and the answer found to be intimately related to the result for cloning: the output is a perfect NOT of the original state $2/3$ of the times and the original state $1/3$ of the times.

Both optimal transformations have been demonstrated [7–9] with near optimal fidelity using stimulated emissions of photons as proposed in Ref. [10]. In two of these realizations [7,8], a source capable of emitting maximally entangled photon pairs into modes a and b was seeded with the photon whose polarization was to be cloned, leading to a two photon state proportional to

$$(\hat{a}_p^\dagger \hat{b}_{\bar{p}}^\dagger - \hat{a}_{\bar{p}}^\dagger \hat{b}_p^\dagger)\hat{a}_p^\dagger|0\rangle = \sqrt{2}|2,0\rangle_a|0,1\rangle_b + |1,1\rangle_a|1,0\rangle_b,$$

where we have introduced the notation $|n,m\rangle_a|l,q\rangle_b = (\hat{a}_p^\dagger)^n(\hat{a}_{\bar{p}}^\dagger)^m(\hat{b}_p^\dagger)^l(\hat{b}_{\bar{p}}^\dagger)^q|0\rangle$; p is the polarization to be cloned and $\bar{p}$ its orthogonal complement. The term in brackets on the left-hand side arises from the creation of a photon pair in modes a and b; the $\hat{a}_p^\dagger|0\rangle$ represents the seed photon whose polarization p is to be cloned. The output (right-hand side) is a superposition of stimulated and spontaneous down-conversion in which mode a contains optimal clones and mode b contains an optimal quantum NOT of the seed photon. The universality arises from the rotational symmetry of the stimulated source, the distortion of the input state from spontaneous emission, and the enhancement of the desired polarization from the stimulation. The entangled nature of the singlet state is necessary only for the implementation of the NOT operation.

The scheme we propose here is based instead on an interference effect at a beam splitter. We require a beam splitter and a source of single photons in a maximally mixed state for the cloning and of photon pairs in a singlet state for the NOT operation. The photon we want to clone (NOT) arrives at the beam splitter via mode c and the mixed state (member of a singlet state) arrives via mode a (see Fig. 1). We carry out the analysis using the entangled state; the result of a mixed state analysis is readily achieved by tracing out the other member of the entangled photon pair at the end of the calculation.

The evolution of the initial state $\frac{1}{\sqrt{2}}(\hat{a}_p^\dagger \hat{b}_{\bar{p}}^\dagger - \hat{a}_{\bar{p}}^\dagger \hat{b}_p^\dagger)\hat{c}_p^\dagger|0\rangle$ is determined by the evolution of modes a and c, $\hat{a}^\dagger \rightarrow (\hat{d}^\dagger + i\hat{e}^\dagger)/\sqrt{2}$, and $\hat{c}^\dagger \rightarrow (i\hat{d}^\dagger + \hat{e}^\dagger)/\sqrt{2}$, giving

$$\frac{1}{2\sqrt{2}}[(\hat{d}_p^\dagger + i\hat{e}_p^\dagger)\hat{b}_{\bar{p}}^\dagger - (\hat{d}_{\bar{p}}^\dagger + i\hat{e}_{\bar{p}}^\dagger)\hat{b}_p^\dagger](i\hat{d}_p^\dagger + \hat{e}_p^\dagger)|0\rangle. \quad (1)$$

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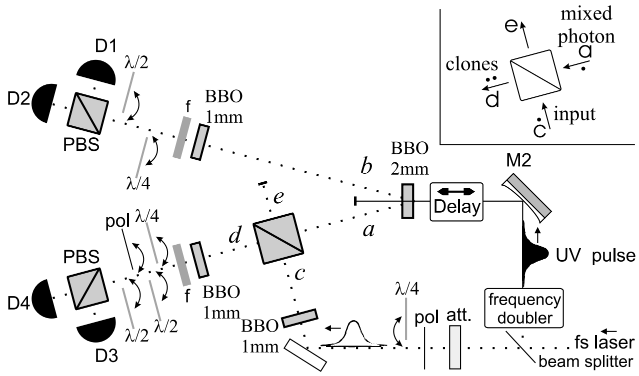

FIG. 1. Apparatus used to demonstrate optimal universal quantum cloning and the optimal universal quantum NOT gate on a beam splitter. A central role is played by the beam splitter with input modes a and c and output modes d and e. One photon of a polarization-entangled photon pair created by pulsed type-II parametric down-conversion inside a 2 mm thick β-BBO crystal enters the beam splitter in mode a. The photon to be cloned enters in mode c.

The terms are naturally divided into three classes according to where the photons go: those quadratic in d (with an associated probability of occurrence p of 3/8), those quadratic in e ($p = 3/8$), and those linear in both ($p = 1/4$). If we select only the cases where they emerge together in mode d we obtain a state proportional to

$$((\hat{d}_p^\dagger)^2\hat{b}_{\bar{p}}^\dagger + \hat{d}_p^\dagger\hat{d}_{\bar{p}}^\dagger\hat{b}_p^\dagger)|0\rangle = \sqrt{2}|2,0\rangle_d|0,1\rangle_b + |1,1\rangle_d|1,0\rangle_b, \qquad (2)$$

which corresponds to optimal clones in mode d and an optimal NOT in mode b. This demonstrates the possibility of performing optimal quantum cloning and the NOT operation on a beam splitter. The universality arises again through the rotational symmetry of the $|\psi^-\rangle$, however, unlike in the scheme based on stimulated emission, the enhancement and distortion arise from postselection of the two-photons-in-one-arm events.

A more intuitive way to see this is to start with the local state of the photon in arm a which is maximally mixed in polarization. This looks the same when expressed in any orthonormal basis; thus we can think of the mixed photon as having, in half the cases, the same polarization as the photon to be cloned and the orthogonal polarization in the remaining half.

When the polarizations are the same, the photons will bunch [11], emerging together in either mode d or mode e:

$$\hat{a}_p^\dagger\hat{c}_p^\dagger|0\rangle \rightarrow (i(\hat{d}_p^\dagger)^2 + i(\hat{e}_p^\dagger)^2)|0\rangle,$$

if instead the two incident photons have orthogonal polarization, there is no interaction, and the photons emerge a quarter of the time both in mode d, a quarter of the time in mode e, and half the time one in d and one in e.

Selecting the cases where the two photons emerge in the same arm makes the probability for the photons to have the same polarization twice as large as that for the photons to have orthogonal polarizations. We can thus immediately write down the polarization state of the photons which emerge in the same arm:

$$\tfrac{2}{3}|2,0\rangle_d\langle 2,0| + \tfrac{1}{3}|1,1\rangle_d\langle 1,1|,$$

where $|n,m\rangle_d = (\hat{d}_p^\dagger)^n(\hat{d}_{\bar{p}}^\dagger)^m|0\rangle$, which is the local output state of an optimal universal cloning machine [2], in agreement with Eq. (2). Using the spin zero nature of the $|\psi^-\rangle$ we can write down the local state of the photon in b:

$$\tfrac{2}{3}|0,1\rangle_b\langle 0,1| + \tfrac{1}{3}|1,0\rangle_b\langle 1,0|.$$

Thus three out of four times, the photons will emerge together in either arm d or arm e as optimal clones of the input polarization; at the same time, if the photon originally in arm a was maximally entangled with a photon in a third arm b, the photon in arm b will be an optimal universal NOT of the original polarization. In the remaining one out of four cases the photon polarization is lost if the photon in arm a were in a mixed state. However, if the photon was maximally entangled with a photon in a third mode b, the polarization is not lost and, in fact, is teleported [12,13] to the photon in b.

Our scheme is thus essentially different from the scheme based on stimulated emission [10], since it achieves the enhancement of the initial polarization by starting out with photons sometimes polarized the same way, sometimes the opposite way, and selectively rejecting cases where they are polarized the opposite way. It is conceptually very simple, economical in its means, and can readily be extended to work for any type of particle with internal degrees of freedom for which a beam splitter can be implemented, such as electrons [14]. Such gain in simplicity and scope for extension is accompanied by a sacrifice in scalability to cloning higher photon numbers; the more photons are sent into the beam splitter, the less likely it becomes to have a successful cloning event.

The apparatus used to demonstrate our schemes experimentally is shown in Fig. 1. The photon to be cloned arrives at the beam splitter through mode c, and the photon it interacts with arrives through mode a. Both modes have the spatiotemporal form of traveling pulses; the use of single mode fibers to couple light into the detectors (D1–D4) guarantees that the photons we see have identical transverse spatial profiles, but their relative time of arrival can be varied. The extent to which they will evolve into the same modes d and e after the beam splitter and thus the strength of their bosonic interaction will depend on their relative time of arrival. The photon in mode b is also in a pulsed mode, and has polarization totally anticorrelated with that of the photon in a.

A Ti:sapphire mode-locked laser produces light pulses of 120 fs duration, centered at a wavelength of 780 nm, with a repetition rate of 82 MHz. Part of the light is frequency doubled; the light which is not doubled is attenuated so that on average only 1 in 80 pulses contains a single photon and its polarization state is prepared as desired. These photons play the role of the photons whose polarization is to be cloned and are sent to input mode c of a 50/50 beam splitter passing first through a birefringent element (1 mm β-BBO) that is there to counter the birefringence of the 1 mm β-BBO farther down the photon path. The frequency doubled light (about 400 mW) travels via a delay line to a 2 mm β-BBO crystal in order to produce entangled photon pairs in the $|\Psi^-\rangle$ state, according to a well-known scheme [15]. A member of each entangled photon pair (mode a) is sent to the other input arm of the 50/50 beam splitter, while the other photon travels toward detectors D1 and D2.

One of the output modes of the beam splitter, mode d, travels through a 1 mm β-BBO crystal, a 5 nm bandwidth filter centered at 780 nm, and the appropriate combination of polarizers, half- and quarter-wave plates to detectors D3 and D4. The crystal and the filter are part of the scheme [15] to create the $|\psi^-\rangle$ state; in our setup the crystal was placed after the beam splitter for reasons of space, making it necessary to place the birefringent element (1 mm β-BBO) in arm c. Coincidence logic allows the selection of threefold events where photons are detected at D3 and D4 and either D1 or D2 within 2 ns of each other. The 2 ns coincidence window is enough to ensure that a threefold event corresponds to a down-converted pair and an extra photon, since the time interval between consecutive pulses is 12 ns. However, it does not guarantee that the photons arriving at D3 and D4 are in the same mode since their relative times of arrival can still differ by much more than the pulse width.

We measure $N(2,0;1,0)$, $N(2,0;0,1)$, $N(1,1;1,0)$, and $N(1,1;0,1)$, corresponding to the number of simultaneous detections of two photons in arm d and one in arm b in states $|2,0\rangle_d|1,0\rangle_b$, $|2,0\rangle_d|0,1\rangle_b$, $|1,1\rangle_d|1,0\rangle_b$, and $|1,1\rangle_d|0,1\rangle_b$. As the delay is varied, $N_d(2,0) = N(2,0;1,0) + N(2,0;0,1)$ is expected to vary between being the same as $N_d(1,1) = N(1,1;1,0) + N(1,1;0,1)$ for a large delay to being twice as large for zero delay, when the modes of the photon to be cloned and the apparatus photon overlap. The operational measure of the quality of cloning is the cloning fidelity, which is the probability that a photon taken from arm d has the polarization of the initial photon in arm c. In terms of the rates, $F = \frac{N_d(2,0)+(1/2)N_d(1,1)}{N_d(2,0)+N_d(1,1)}$. For the optimal $N_d(2,0)/N_d(1,1) = 2$, this gives $F = 0.83$. For the NOT operation we have $F = \frac{N_b(0,1)}{N_b(1,0)+N_b(0,1)}$, where $N_b(1,0) = N(2,0;1,0) + N(1,1;1,0) + N(0,2;1,0)$ with an optimal $F = 0.66$.

The results for cloning can be found in Fig. 2. The number of events $N_d(2,0)$, where two photons in d were projected onto the input polarization, increases as zero delay is approached, while the number of events $N_d(1,1)$, where one of the photons was projected onto the input polarization and the other onto its orthogonal complement, remains the same. The discrepancy between $N_d(2,0)$ and $N_d(1,1)$ for large delays arises primarily from the fact that in order to measure $N_d(2,0)$ in our setup it was necessary to use a polarizer, reducing the counts by a factor of 2, and from the fact that the graphs were taken at different times under slightly different conditions. To calculate the fidelity, we take, from a Gaussian fit to $N_d(2,0)$, the ratio of $N_d(2,0)$ at zero delay to $N_d(2,0)$ in the flat region as a faithful measure of the ratio $N_d(2,0)/N_d(1,1)$ at zero delay. Thus we obtain the fidelities from the ratio of peak to background counts.

The values obtained for the graphs in Fig. 2 are $F = 0.81 \pm 0.01$ for the linear 0°, $F = 0.80 \pm 0.01$ for the linear 45°, and $F = 0.80 \pm 0.01$ for the left circular polarization. We also cloned linear 90°, −45°, and right circular obtaining similar results. The main reason the values differ from the best achievable ones is that input modes a and c have different pulse shapes even after passing through 5 nm bandwidth interference filters. (The singlet source also produces four-photon states. This leads to the possibility of recording a threefold coincidence even when no photon was originally present in arm c, giving a level of counts that does not depend on the delay, reducing the visibility. For our parameters this contributes for only one part in 100.) The photon in mode a is generated by simply attenuating part of a pulse from the mode-locked laser and thus arrives in the form of a transform limited pulse. The profile of the photon in mode c is instead dictated by the details of the nonlinear interaction in the β-BBO crystal where it is produced. A calculation in the spirit of [16] predicts the observed fidelities of 0.81.

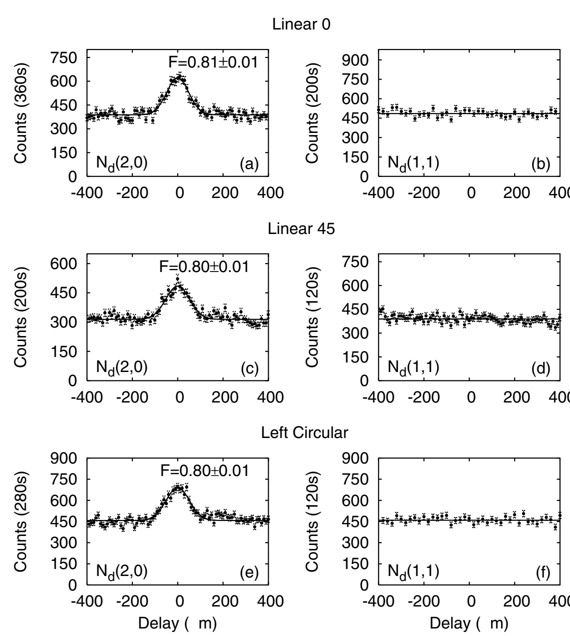

FIG. 2. Experimental data demonstrating cloning on a beam splitter. $N_d(2,0)$ increases as the photon wave packets overlap, while $N_d(1,1)$ remains the same. The data on the left are fitted to a Gaussian and the fidelity, displayed in each plot, is deduced from the fit as described in the text.

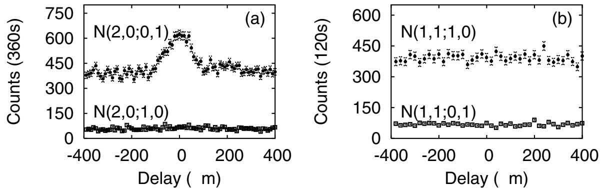

FIG. 3. Experimental data demonstrating the NOT operation of circularly polarized light. Similar results were obtained for all other polarizations attempted (linear 0°, 90°, 45°, −45°).

Figure 3 shows the measured $N(2,0;0,1)$, $N(2,0;1,0)$, $N(1,1;0,1)$, and $N(1,1;1,0)$ for circular polarization. $N(0,2;1,0)$ and $N(0,2;0,1)$ were found to be zero. Similar assumptions about the rates in different runs to those made for the cloning lead to a NOT fidelity of 0.60 ± 0.01. The value differs from the ideal one both because of the mode-distinguishability effects discussed above and because of a lack of purity in the $|\psi^-\rangle$ state produced in our setup. The absolute number of counts for both experiments is determined by the repetition rate (82 MHz), the probability of getting a down-converted pair per pulse, the overall incoupling and detection efficiency (10% per detector), and the requirement that the probability of getting two photons in the input mode c be low.

It is interesting to note that our scheme, together with schemes for teleportation [13], entanglement purification [17], entanglement swapping [18], partial bell state projection [19], and quantum computation [20] makes indistinguishability effects induced at a beam splitter one of the most versatile tools for the practical implementation of quantum information.

Before concluding, we point out how to generalize our scheme to particles with spin 1/2 and to clone more than optimally given some knowledge about the state. For particles with spin 1/2, for example, electrons, the bunching would be replaced by antibunching because they obey the Pauli exclusion principle. The "good" events simply become those where one particle emerges in mode d and one particle in mode e; otherwise the scheme is identical. Another generalization of our scheme is to clone more than optimally polarizations that are not completely unknown, in particular, if the state to be cloned is known to lie within a closed area of the Poincaré sphere. In such a case, changing the state of the maximally mixed state biased in favor of the direction which is the geometric mean of the closed area discussed above will increase the chances that the photons have the same polarization to start with, increasing in turn the fraction of the photons that bunch and thus the cloning fidelity.

In conclusion, we have proposed a scheme to implement both optimal $1 \rightarrow 2$ quantum cloning and the optimal universal quantum NOT operation using a beam splitter. The scheme works for all particles of fractional and integer spin for which a beam splitter can be implemented. We demonstrate the scheme with photons. Finally, we suggest a way in which the scheme can be generalized to carry out more than optimal cloning given partial information on the state to clone. A natural extension of the work would be to investigate linear optical implementations of $N \rightarrow M$ optimal quantum cloning.

During the experimental stages of this work, we became aware of a parallel research effort [21].

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