Transitions in viscous withdrawal

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Minisymposium:
Tip-streaming & flow/EHD flow focusing

APS-DFD Meeting
San Antonio, TX
November 2008
Introduction: flow-driven shape transition

selective withdrawal   viscous entrainment

above $Q_c$ threshold withdrawal flux

1 cm

hump   spout

Simple example of non-equilibrium transition?

Cohen & Nagel PRL 2002
Thin spout used to encapsulate biological cells for transplant therapy

immunoisolation of pancreatic Islets

Jason Wyman, Seda Kizilel, Milan Mrksich, Sidney R. Nagel, Marc R. Garfinkel, …, University of Chicago

- Bigger particles initiate breakup in thicker region of spout
- Thin, uniform coat on irregular particles
- Independent of particle size
Background

Axisymmetric withdrawal in stratified, deep layers

1. Selective withdrawal / viscous entrainment  
   Cohen PRE ‘04, Zhang PRL ‘04, Case & Nagel PRL ‘07, Blanchette & Zhang ‘08…

2. Tendril in geophysical flows  
   Ivey & Blake ‘85, Lister ‘89; Davaille ‘99, Jellinek & Manga ‘02, Davaille, Girard & Le Bars ‘02, Schmidt & Zhang ‘08…

3. Viscous drainage  
   Chaieb ‘04, Courrech du Pont & Eggers ‘06…

Related phenomena

1. Flow-focusing  
   Ganan-Calvo et al. ‘00, Anna, Bontourx & Stone ‘03, Utada et al. ‘05, Suryo & Basaran ‘06, Marin et al. ‘07…

2. 2D air entrainment  
   Joseph et al. ‘91, Jeong & Moffatt ‘92, Simpkin & Kuck ‘00, Eggers ‘01, Lorenceau, Restagno & Quere ‘03…

3. Drop emulsification  
   Taylor ‘34, Taylor ‘64, Buckmaster ‘73, Rallison & Acrivos ‘78, Acrivos & Lo ‘78, Grace ‘82, Stone ‘94, Navot ‘99, Blawzdziewicz, Cristini & Loewenberg ‘02…

4. Inviscid drainage  
   Sautreaux ‘01, Lubin & Springer ‘67, Imberger & Hamblin ‘82, Tuck & Vanden-Broeck ‘84, Miloh & Tyvand ‘93, Hocking & Forbes ‘01…
**Hump spout transition**

How does the surface fail?
- hump height $h_c$
- curvature at top of hump $k_c$

When does the surface fail?
- threshold volume flux $Q_c$

Focus on evolution of steady-state solution
*Not dynamical transformation between 2 states*
Experiment’s parameters

- **Pipette Height**  $S \sim 0.5 \text{ cm}$
- **Capillary Length**  $l_i \sim 0.3 \text{ cm}$
- **Strain Rate**  $E \sim Q/4$  $S^3 \sim 0.3 \text{ s}^{-1}$
- **Hump Height**  $h \sim 0.1 \text{ cm}$
- **Hump Curvature**  $\# \sim 1/(50 \mu \text{m})$

\[ \text{Re} = \text{inertia} / \text{viscous} \quad 0.006 \]
\[ \text{Ca} = \text{viscous} / \text{surface tension} \quad 0.02 \]
\[ \text{Bo} = \text{hydrostatic pressure} / \text{surface tension} \quad 0.1 \]
\[ \lambda = \text{lower layer viscosity} / \text{upper layer viscosity} \quad 1 \]
Minimal model
Marko Kleine Berkenbusch & Itai Cohen

• Deep lower layer modelled as interior surface with constant pressure jump
• Focus on $S < \text{Capillary length } l_\text{c}$
  Pin surface at $r=a$
  (Lister JFM ‘89 analyzes $S >> l_\text{c}$ regime)

Kleine Berkenbusch, Cohen & Zhang JFM in press
Numerical results: steady shape

Increasing withdrawal flux $Q$
Numerical results: steady shape

increasing withdrawal flux $Q$

smooth shape @ $Q_c$
Numerical results: steady shape

increasing withdrawal flux $Q$

Above $Q_c$, finger grows until it reaches sink

smooth shape @ $Q_c$
Numerical results: steady shape

Above $Q_c$, finger grows until it reaches sink
smooth shape @ $Q_c$

increasing withdrawal flux $Q$
Numerics as $Q \rightarrow Q_c$

height saturates as $(Q_c - Q)^{1/2}$

$$h_c \uparrow$$

curvature slowly saturates as $(Q_c - Q)^{1/2}$
Experiment as $Q_c$  

height saturates 
as $(Q_c - Q)^{1/2}$
**Experiment as $Q$**

$Q_c$

**height saturates as** $(Q_c - Q)^{1/2}$

Together numerics & measurement show curvature saturates

**curvature just starting to saturate at end of range**
How does the surface fail?

Steady surface fails “everywhere at once”
(saddle-node bifurcation)
**Hump spout transition**

How does the surface fail?

*Steady surface fails “everywhere at once” (saddle-node bifurcation)*

When does the surface fail?

*Threshold volume flux $Q_c$*
Numerics: two layers of different viscosities

Francois Blanchette

(\frac{\text{lower layer viscosity}}{\text{upper layer viscosity}}) \quad \mu_0/\mu

\textit{insensitive to viscosity contrast}
Steady-state shapes at $Q_c$
Steady-state shapes at $Q_c$
Recall linear stability  Lister JFM ‘89

Downward force by surface tension is essential for steady hump solution
(resistance from gravity & lower-layer flow are NOT enough)

→ Focus on contributions to surface tension force
Estimate force due to surface tension

contribution from sharply curved tip

\[ F_{\text{tip}} \sim (2 \#_c) \left[ 2 \left( \frac{1}{\#_c} \right)^2 \right] \sim 4 \frac{1}{\#_c} \]

Laplace pressure

Overall deflection dominates if \( h_c \#_c >> 1 \)

contribution from overall deflection

\[ F_{\text{defl}} \sim \left( \frac{2}{l} \right) (h_c l) \sim 2 \frac{1}{h_c} \]

deflected area

Laplace pressure