4.1 Energy current and the Poynting vector  This problem is intended to integrate things you covered last quarter with field energy concepts covered this quarter.

A V-volt battery is connected to a resistor of resistance $R$ via a long, straight coaxial cable. The inner conductor is a cylindrical shell of radius $a$; the outer cylinder has radius $b$. The battery and the cable have negligible resistance.

a) What is the current $i$?
b) Find the direction and magnitude of the Poynting vector $S$ at distance $r$ from the axis of the cylinder, for $r < a$, $a < r < b$, and $b < r$.
c) Find the flux of $S$ along the axis: $\int S \cdot \hat{n} da$, where $\hat{n}$ points along the axis.
d) Relate this flux to the rate of energy dissipation in the resistor.

4.2 transformation of velocity  A particle moves from the origin to the point $(x, t)$ in one co-ordinate system. Assume $x$ and $t$ are both positive. An observer moving to the right at a slow speed $u$ observes this same particle as moving from the origin to $(x', t')$. If $x/t \ll c$ there should be no relativistic effects and we should have $x'/t' = x/t - u$. However, relativity says that if $x/t = c$ then $x'/t'$ must be $c$ also. Thus, the bigger $x/t$ is, the less it is affected by the change of observer.

a) For what value of $x/t$ will we have $x'/t' = x/t - \frac{1}{2}u$? Since $u$ is small (compared to $c$), this can be done using the properties of infinitesimal Lorentz transformations derived in class.

4.3 thinking through kinematics  One type of cosmic ray detector is a rectangular piece of material that gives an electrical signal when a cosmic ray passes through it. Two such detectors are set a distance $a$ apart (the $x$ in the figure should be an $a$). The one on the left detects a cosmic ray at time $t = 0$; at time $b$ later, the detector on the right happens to detect a second ray (not shown in the picture). ($a$ is much larger than the size of the detectors.) A similar pair of detectors is traveling to the right at speed $u$, just above the first pair. The moving detectors also detect the two cosmic rays.

a) Use a pair of reference frames which have $x = t = x' = t' = 0$ at the site of the first cosmic ray detection. Using the Lorentz transformation, find the time interval between the two rays as observed by the moving detectors.
b) The time dilation formula says $t' = \gamma t$. This does not agree with a). Which formula is right? Why?
c) By what distance must the moving experimenter separate his two detectors if they are to detect the same pair of rays?
d) Is it possible for the moving pair to move in such a way that paddle on right (for either observer) triggers first rather than the left one? What restrictions on $a$ and $b$ make this possible? If these restrictions are met, what values of $u$ will give this result?