5.1 doppler shift and four momentum  The frequency $f$ of a wave is the inverse of the time interval between two crests. An sinusoidal electromagnetic wave is emitted from a the origin of the unprimed co-ordinate system, with frequency $\omega$. The primed co-ordinate system is moving in the $x_1$ direction with speed $v$.

a) What is the frequency observed in the primed frame for the waves traveling in the $x_1$ direction? The $-x'_1$ direction?

b) What is the frequency observed in the primed frame for the waves traveling in the $x'_2$ direction?

c) A photon’s four-momentum $p$ is a null vector whose spatial components point in the direction of propagation. Its zero component is proportional to the frequency $f$: $p_0 = hf$, where $h$ is a fundamental constant. By transforming $p$ to the primed frame, verify that $p'_0 = h f'$ where $f'$ are the frequencies found in a) and b).

5.2 relativistic collision cf. Griffiths 10.30  A particle of mass $m$ whose total energy is twice its rest energy collides with an identical particle at rest. If they stick together, what is the mass of the resulting composite particle? What is its velocity? Use the conservation of 4-momentum.

5.3 colliding beams, cf. Griffiths 10.32  Suppose present-day accelerators can accelerate particles of mass $m$ to an energy $E = \gamma mc^2$.

a) What is the center of mass energy when such a particle collides with a like particle at rest?

b) Now suppose two oppositely-moving particles with this same energy collide. What is the center of mass energy? Compare with a).

5.4 angle distortion  An observer moving at speed $u$ sends out a ray of light at an angle $\theta$ from her direction of motion. An observer at rest measures the direction $\theta'$ of this ray.

a) What is the relation between $\theta$ and $\theta'$?