The physics of granular materials

F. Melo

Universidad de Santiago de Chile
and
CIMAT

Collaborators:
Leonardo Caballero
Stephan Job
Francisco Vivanco
Eugenio Hamm

Chicago, Octuber 2004

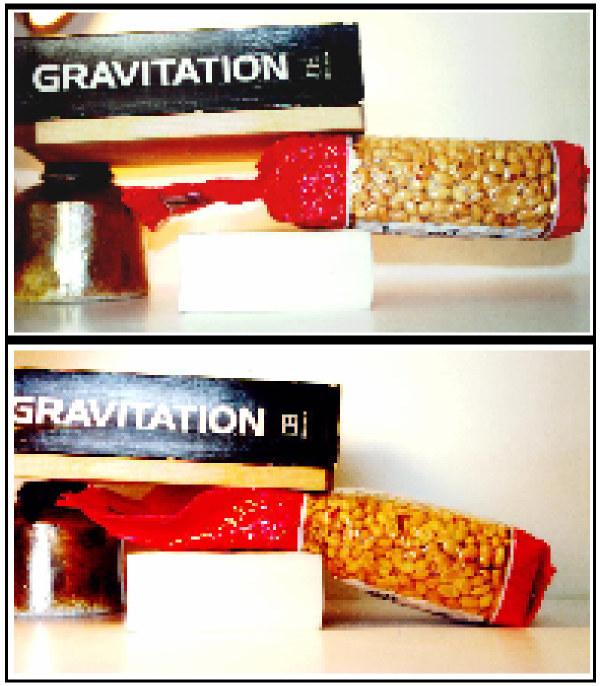
From a grain to avalanches: Exhibit at the Science museum of Santiago

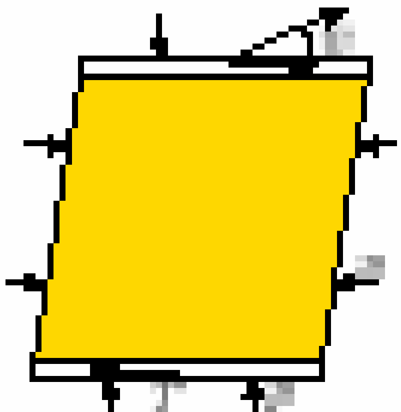
- -Dilation.
- -Hopper flows.
- -Waves in vibrated layers.
- -Droplets of fine grains: wetting?
- -Sound in chains of beads.

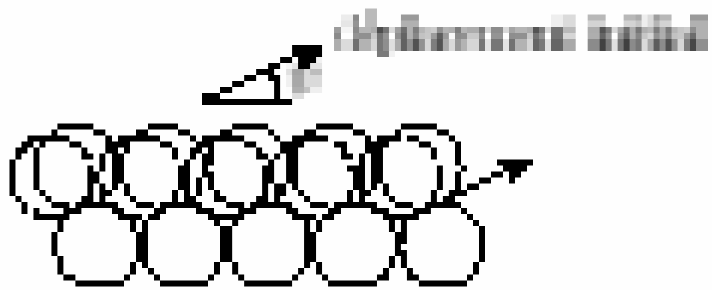
Newton, Hertz, Faraday, Reynolds, Bagnold.

Ottino, Nagel, Jaeger, Umbanhowar and many others.

Dilatancy in a granular material: *Reynolds 1885*

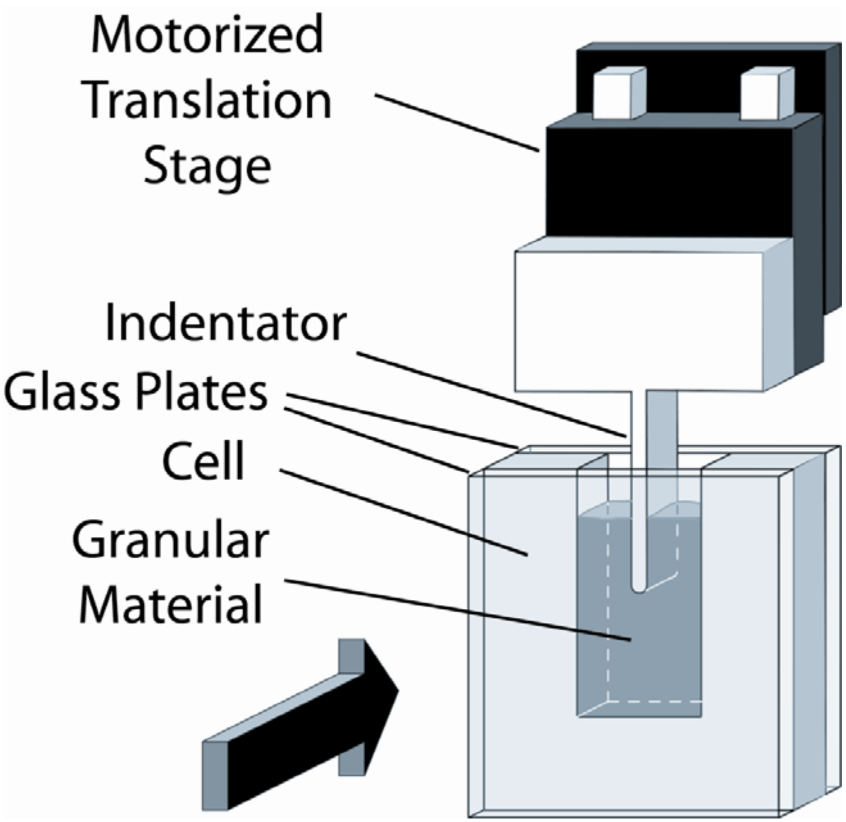




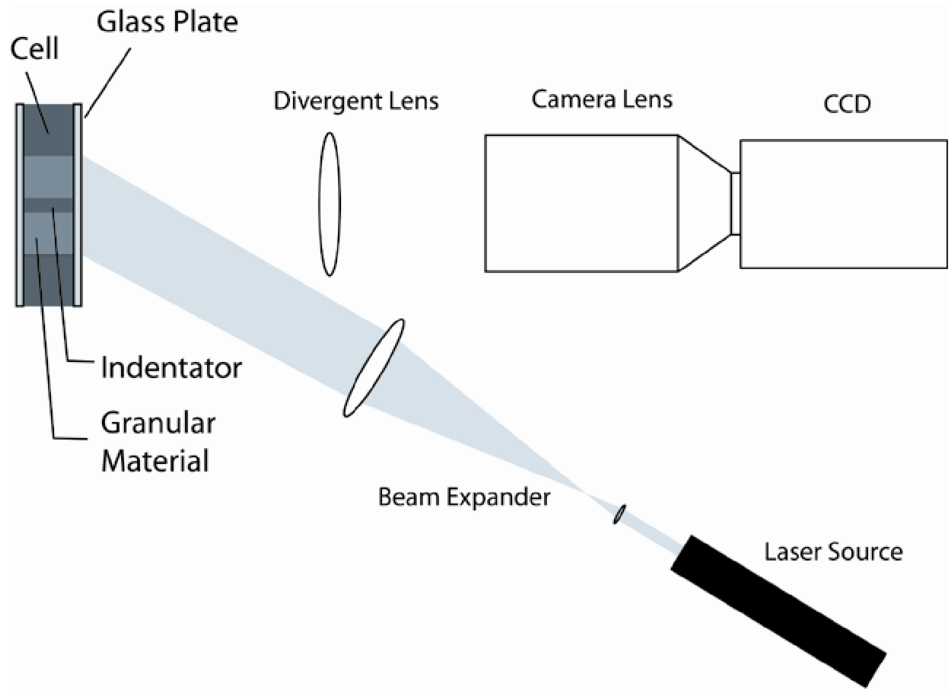


This explain what happens when walking on wet sand.

Walking on sand: a model experiment



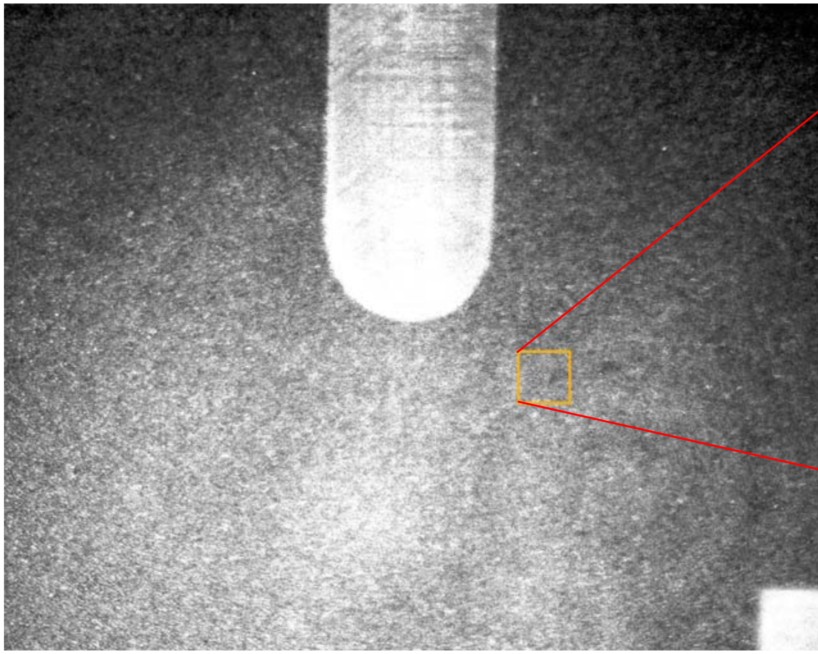
Speckles methods

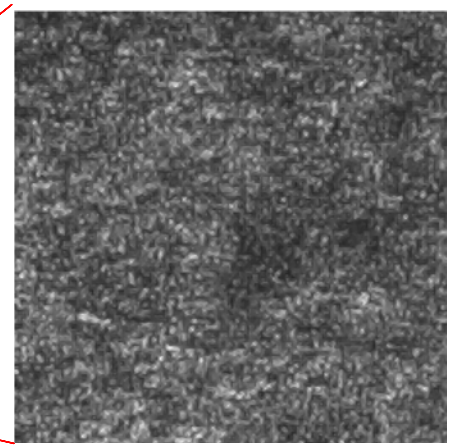


Cell size: 1cm deep, 2cm wide, 5cm long

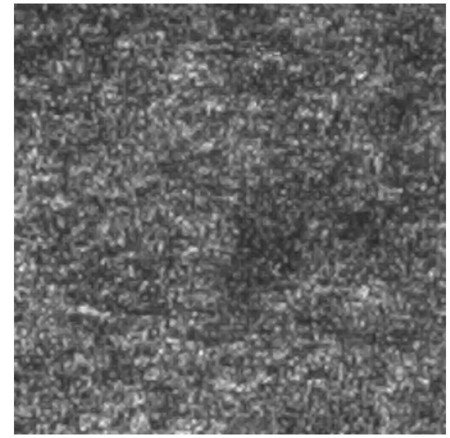
Pushing object: 3mm

Speckles method for displacements

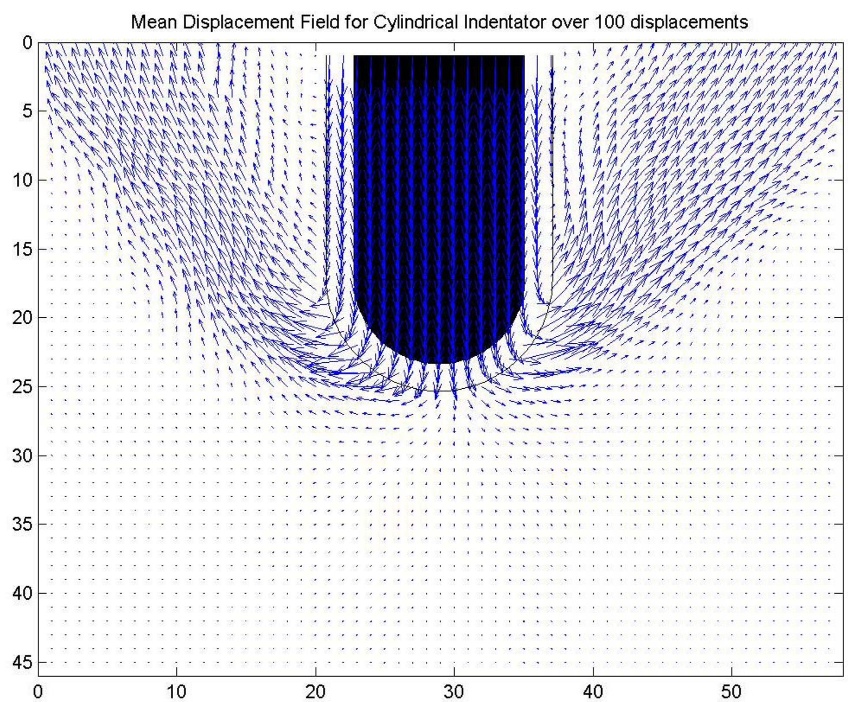




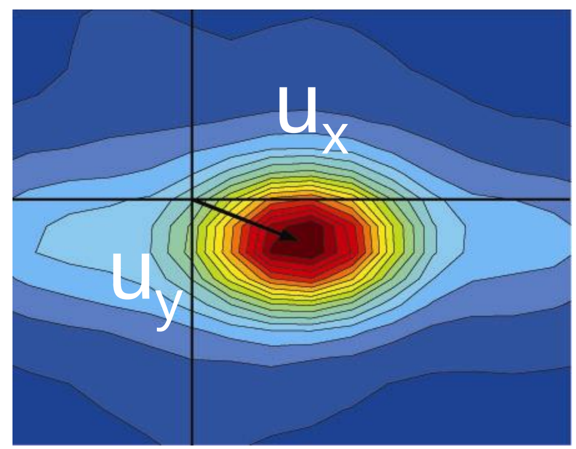
Before displacement



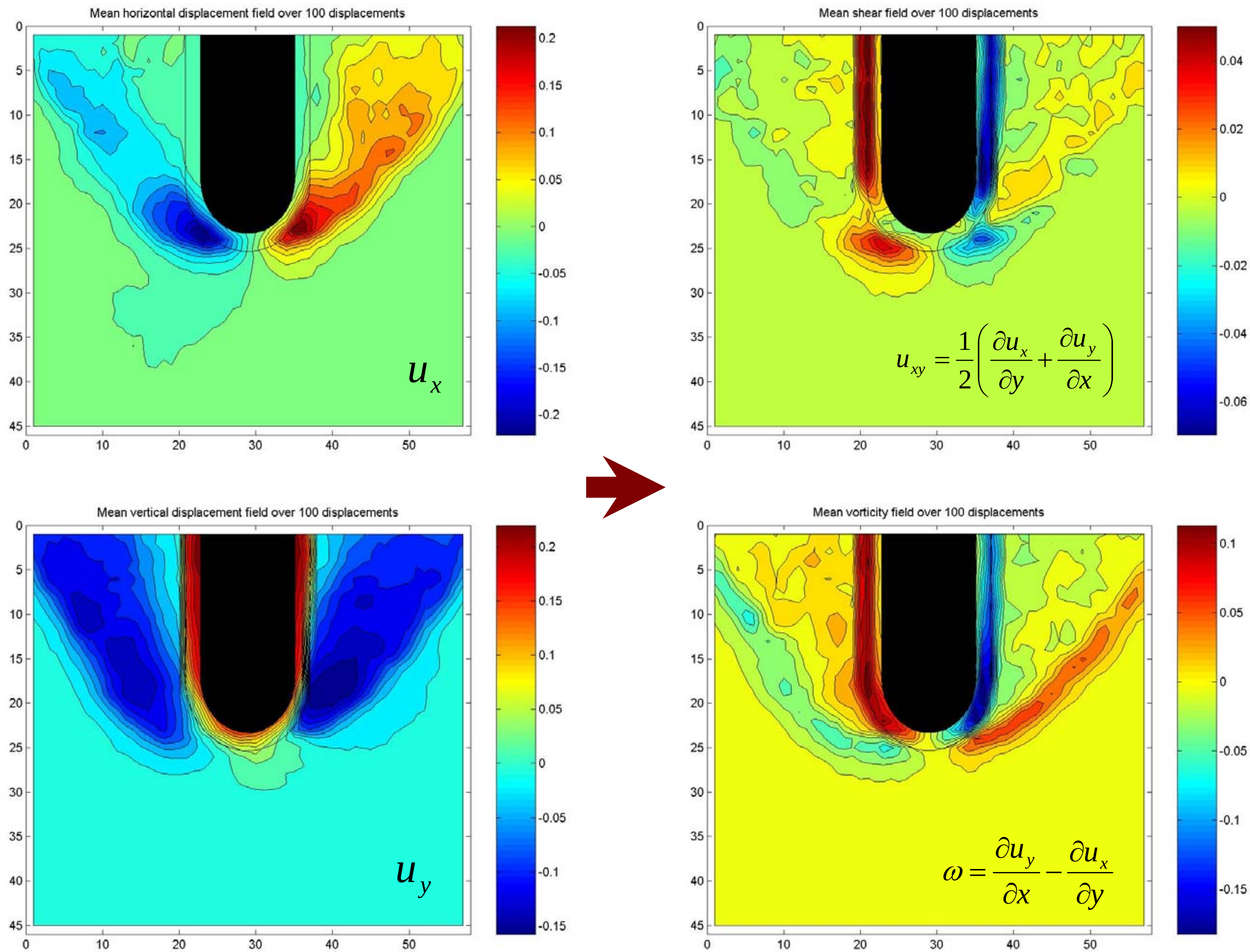
After



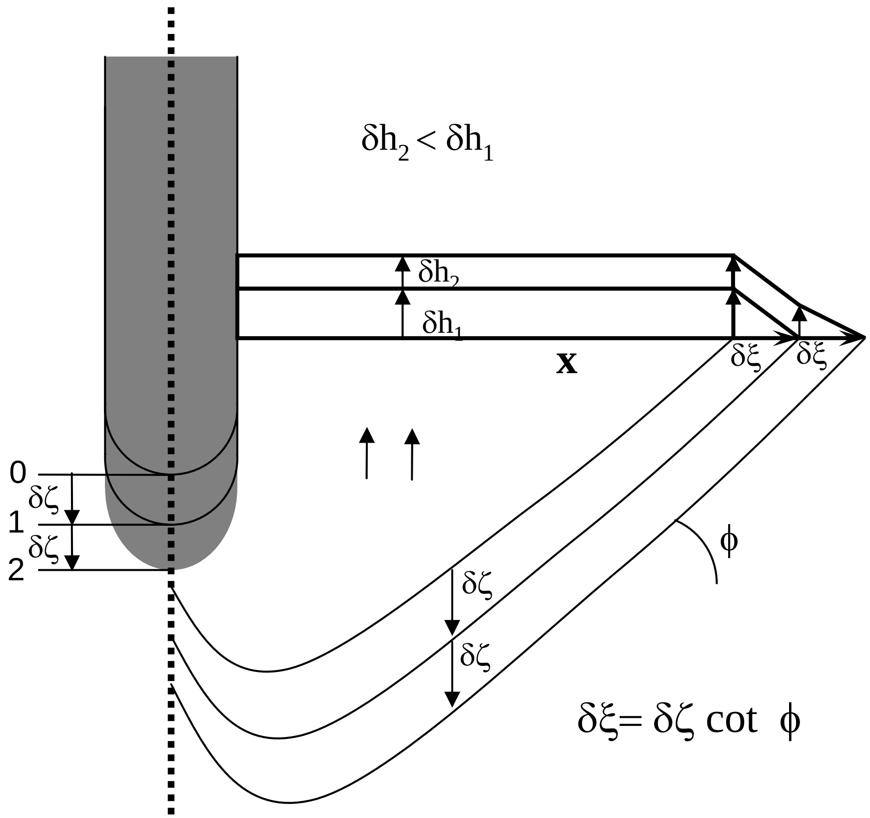
Correlation



Scanning


Mean horizontal displacement field over 100 displacements
u_x
Mean shear field over 100 displacements
$u_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)$
Mean vertical displacement field over 100 displacements
u_y
Mean vorticity field over 100 displacements
$\omega = \frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}$

A rough estimate of the free surface deflection.



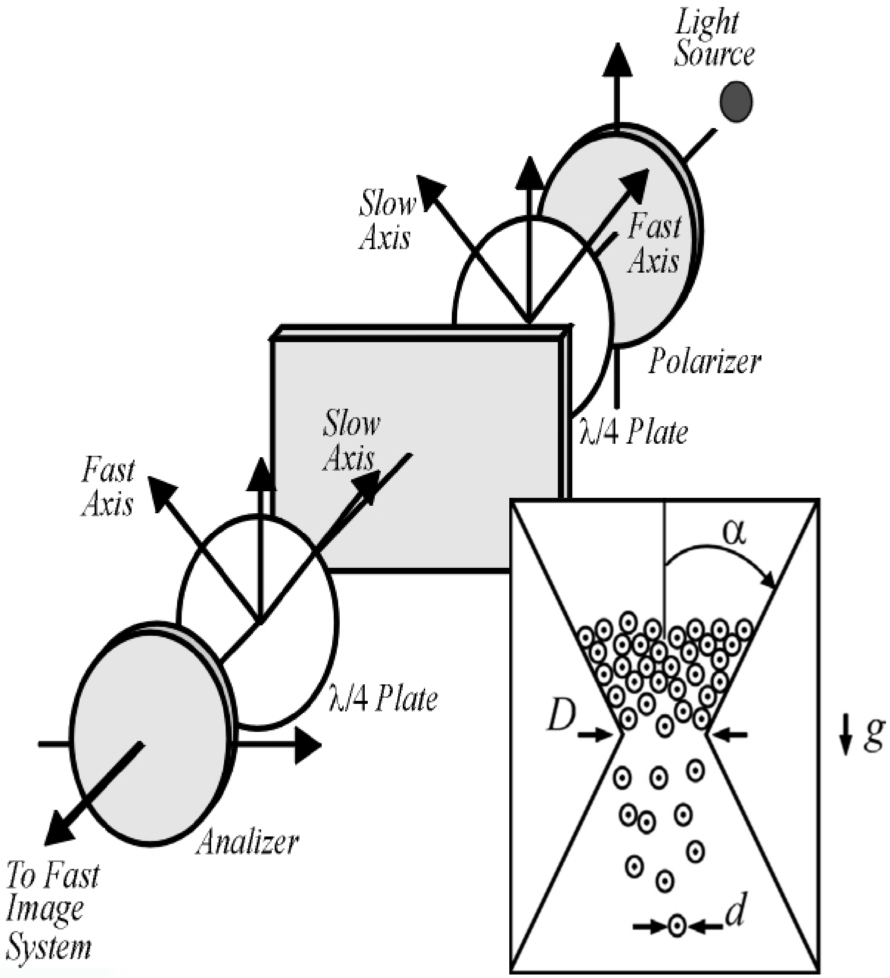
Mass conservation:

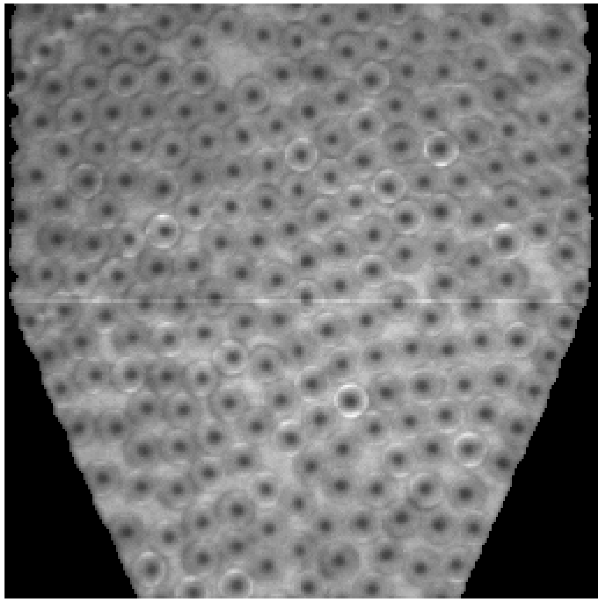
$$\delta h = \frac{R}{\xi - R}\delta\zeta$$

$$h(x,\xi) = Cte + R\tan\phi\log\left(\frac{\xi - R}{x - R}\right)$$

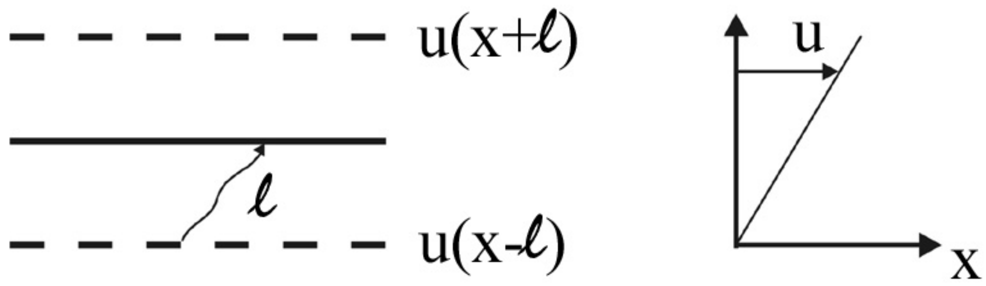
Check next vacations!.

Hopper flows: How hourglass works





In a fluid



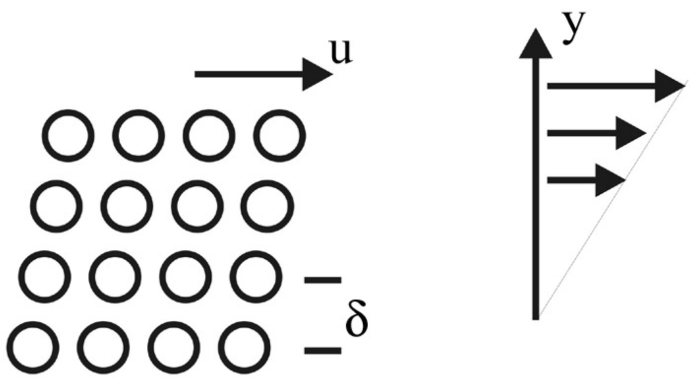
vertical transfer of momentum/area time

$$\sigma \sim \frac{1}{6} n\bar{v}\left[mu(x+l) - mu(x-l)\right] \qquad \sigma = \frac{1}{3} n\bar{v}ml\left(\frac{du}{dx}\right)$$

$$\boxed{\eta = \frac{1}{3} n\bar{v}lm}$$

“Thermal speed” ~ c

In a granular material



$$\sigma \sim \frac{\rho_p D^3}{(\delta + D)^2} \frac{\Delta u}{t_c}$$

$$\frac{\rho_p D^3}{(\delta + D)} \frac{1}{t_c} \left(\frac{\Delta u}{\delta + D} \right) \sim \frac{\rho_p D^3}{(\delta + D)} \frac{1}{t_c} \left(\frac{du}{dy} \right)$$

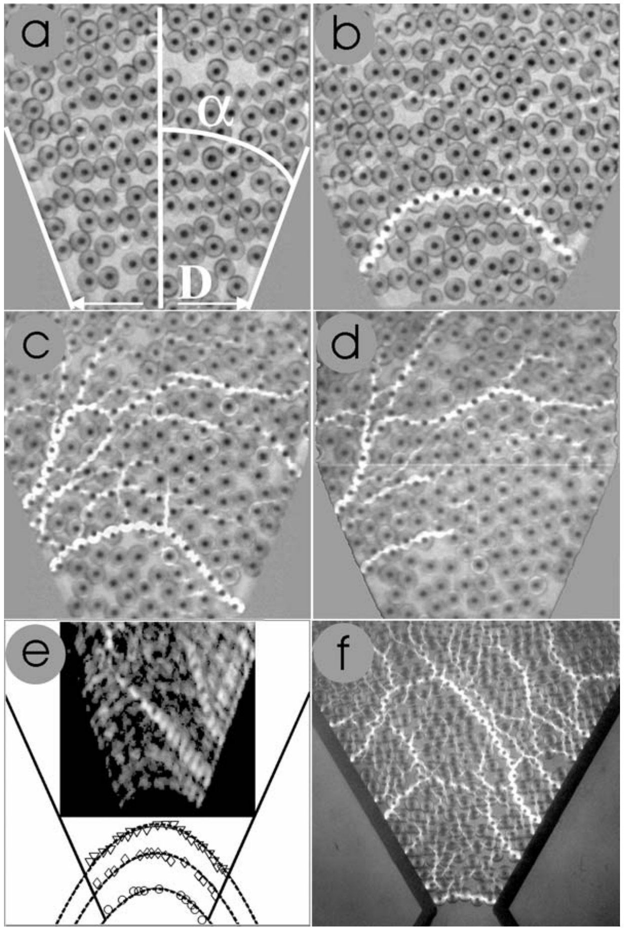
$$t_c \sim \frac{\delta}{\bar{v}} \qquad \sigma \sim \frac{\rho_p D^3}{(\delta + D)} \frac{\bar{v}}{\delta} \frac{du}{dy}$$

but

$$\bar{v} \sim \Delta u \Rightarrow \sigma \sim \left(\frac{du}{dy} \right)^2$$

Bagnold 1941

Force fluctuations might dominate flows



In general there is no intrinsic thermal speed:
-Poor separation scale: Hydrodynamic difficult
-Random fluctuations depend on energy injection
-Absence of well adapted experimental methods.

-Free falling arch?

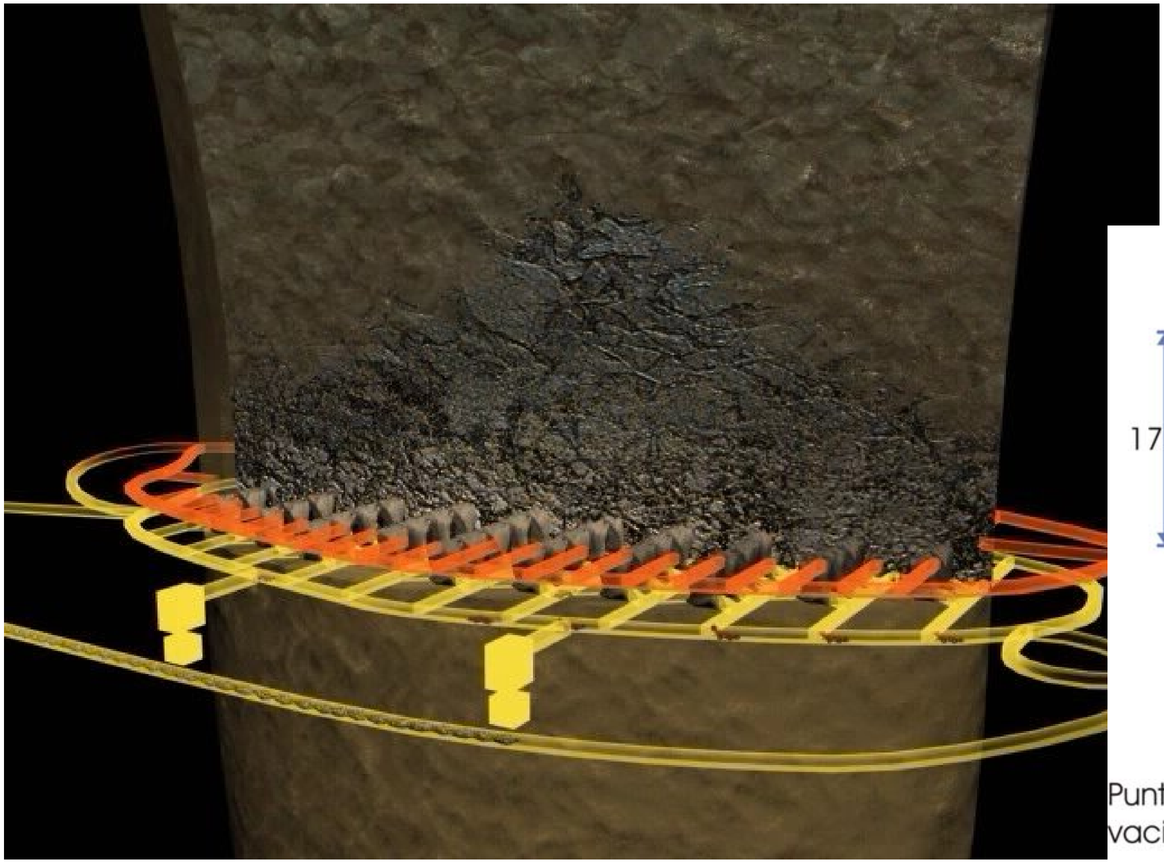
-Grains accelerate over a distance D:

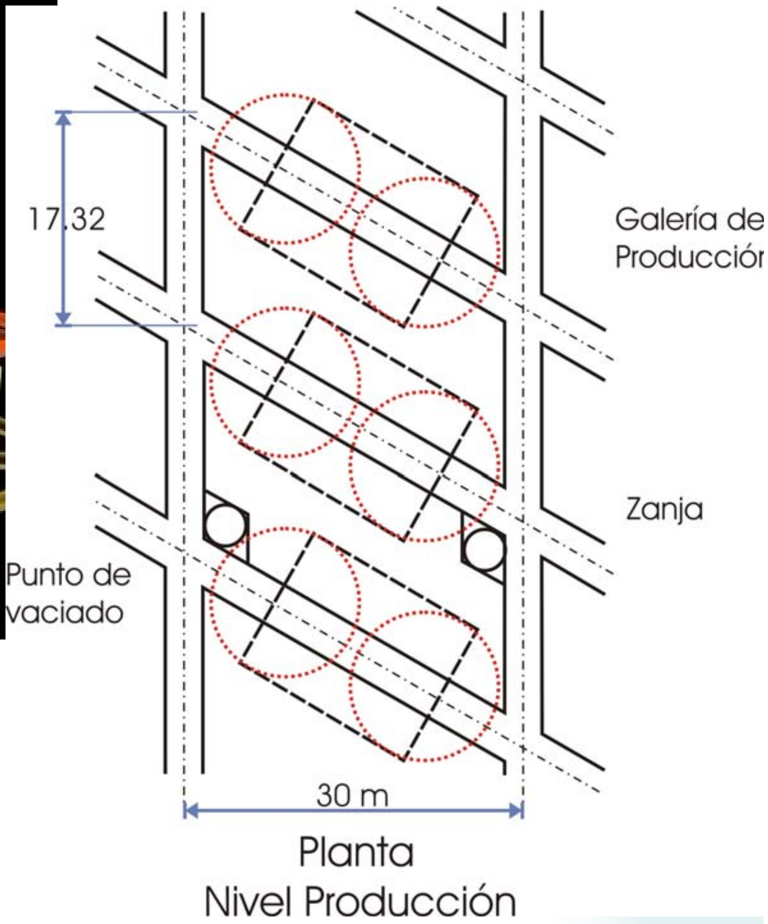
$$V \propto \sqrt{gD}$$

For a fluid $V \propto \sqrt{gH}$

For granular materials $V \propto \sqrt{gD}$

Applications I: Underground copper mining
Chile largest copper producer





The standard procedure for underground mining

Cut hoppers

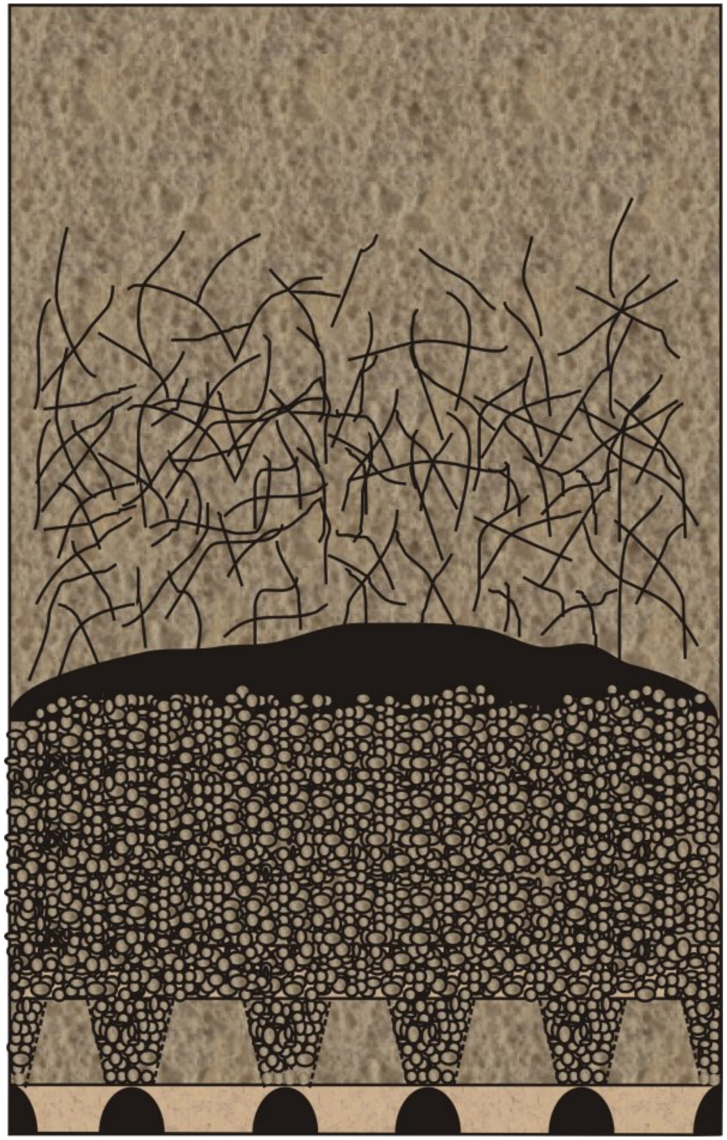
The starting point

"Induce fracture initiation"

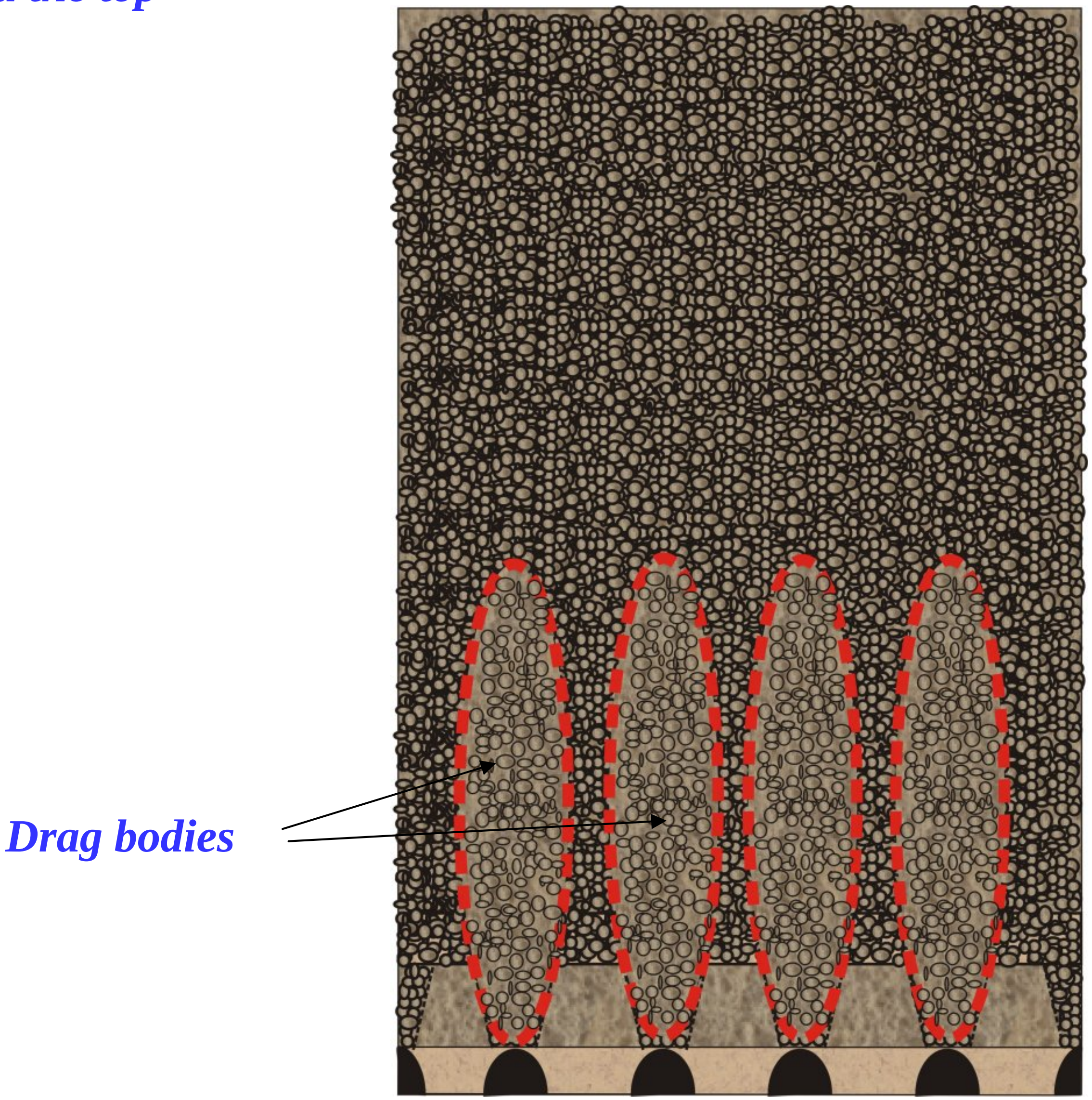
The common believe

As the mineral is extracted the fracture front propagates…

If the mineral is extracted too fast, instable cavities might form.



“When 30% of the mineral has been extracted, the fracture front has reached the top”



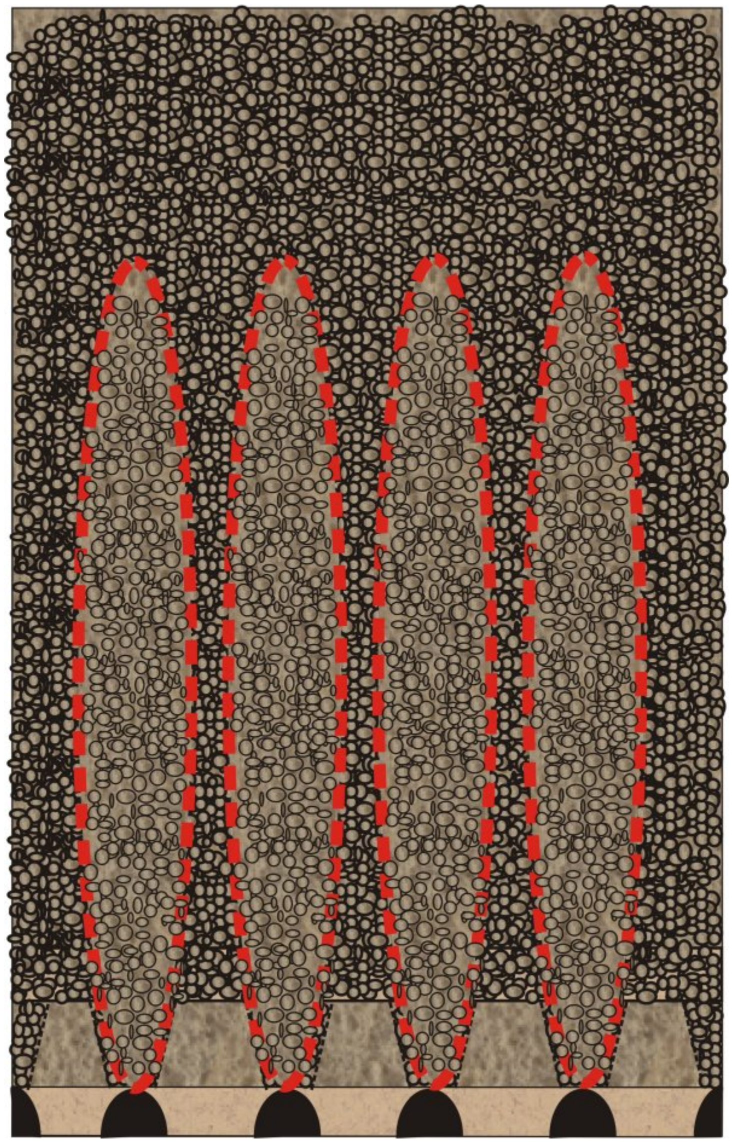
Important questions:

-How to avoid extracting poor mineral at the top.

-How drag bodies evolve.

-How drag bodies interact.

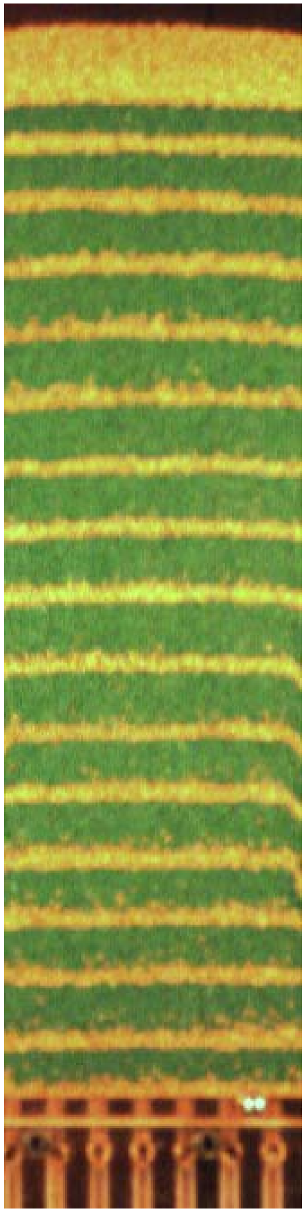
-How to optimize drag bodies size.



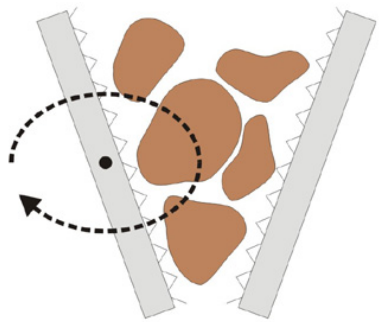
Scientific basis to make decisions!! Avoid common believes.

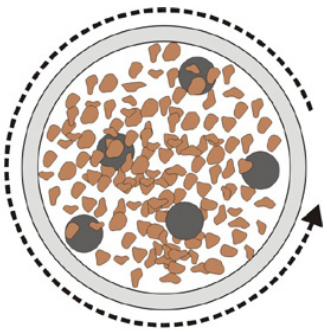
Modeling drag bodies interactions

Valves

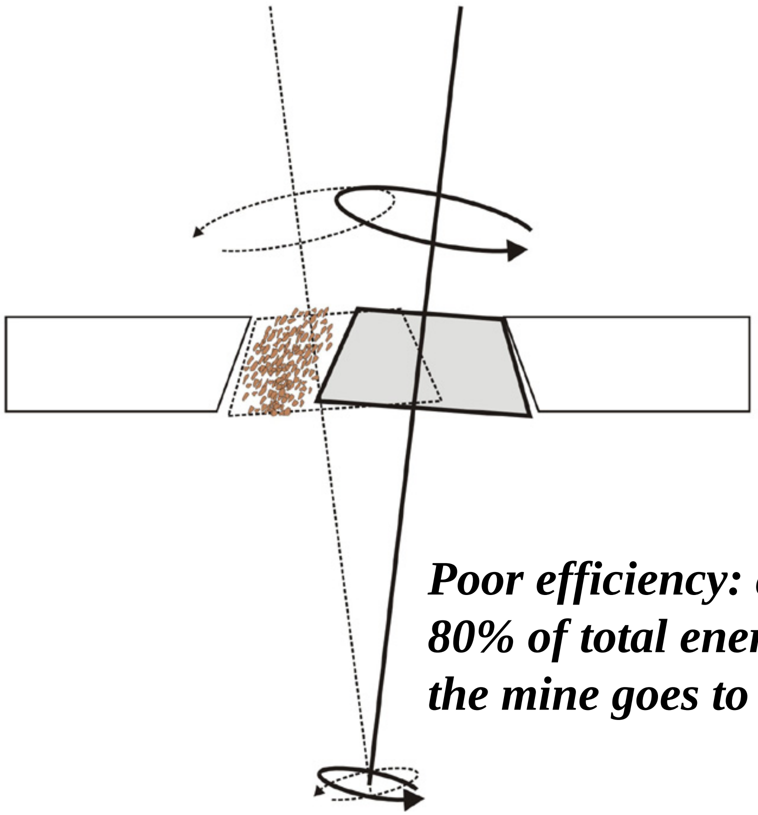


Applications II





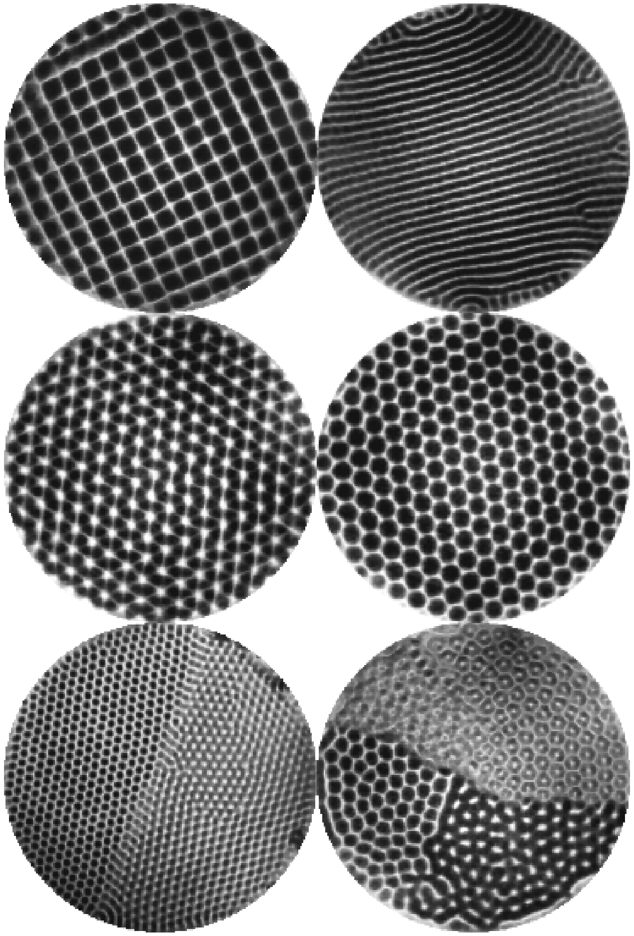
-Grinding machine.
-Flow optimization.
-Size selection.



Poor efficiency: about 5%.
80% of total energy requirements in the mine goes to grinding processes.

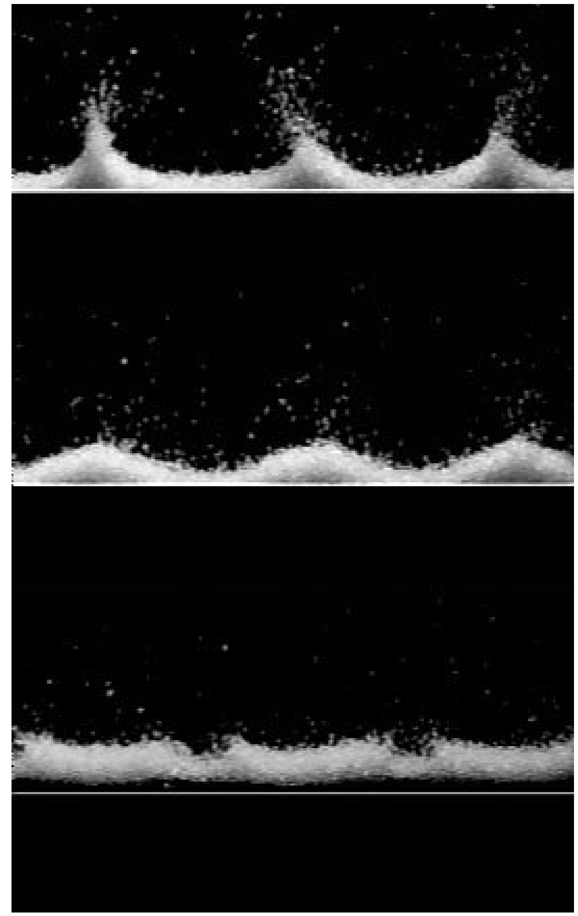
Vibrated granular materials

Top view



Surface Waves on the granular layer:
Fluid like behavior (Umbanhowar, Swinney)

Side view



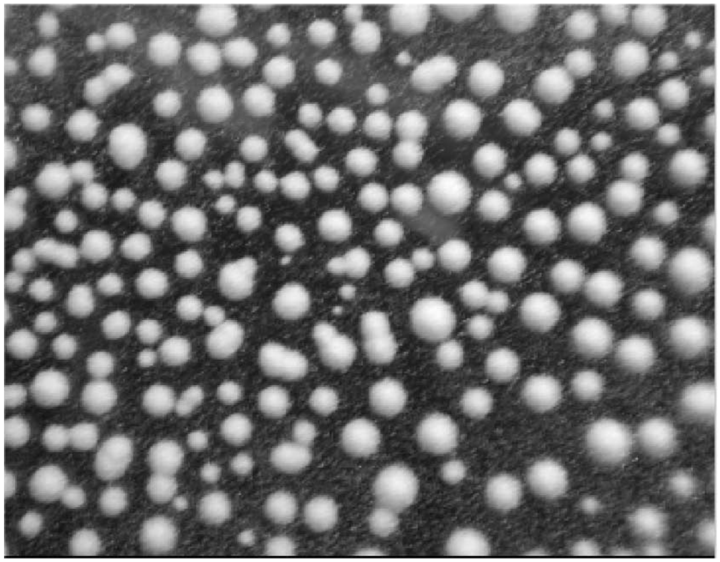
Parametric instability at f/2

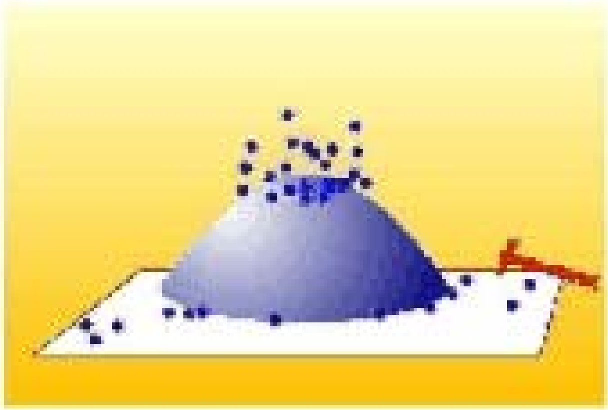
CIMAT

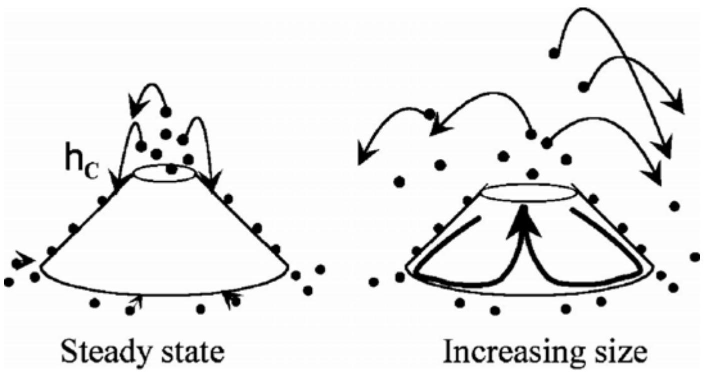
Fine Powders: air effects

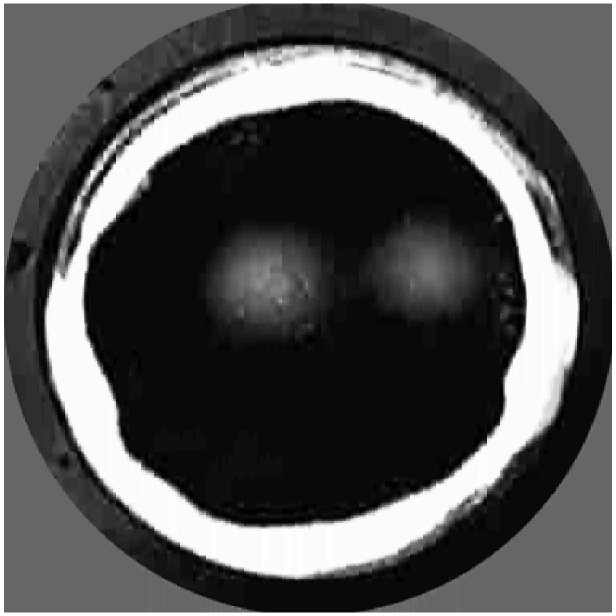
USACH

A cartoon, J. Duran

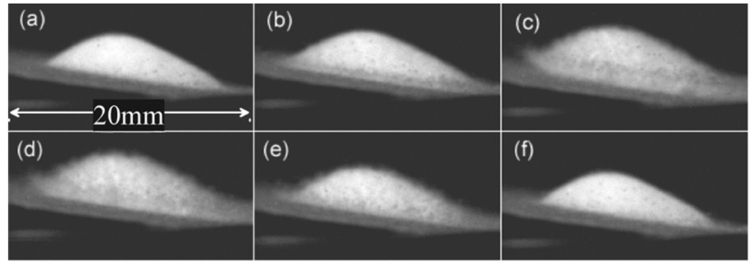


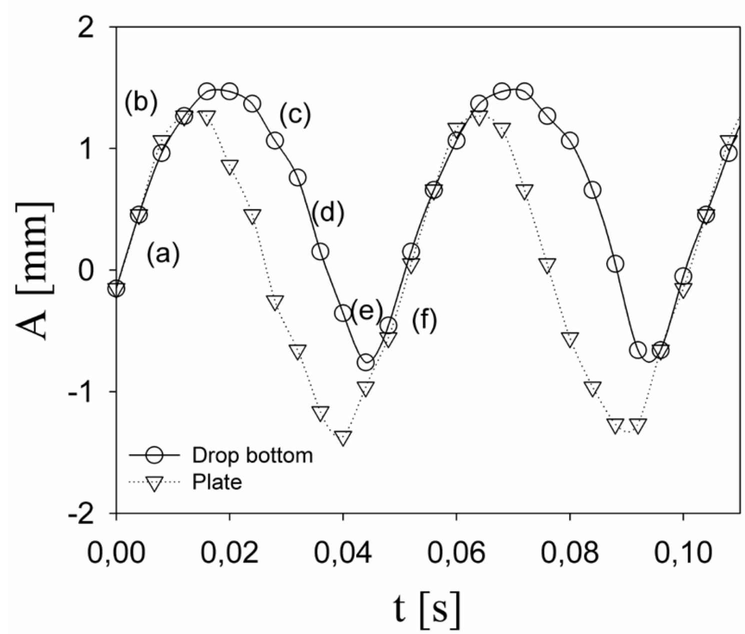






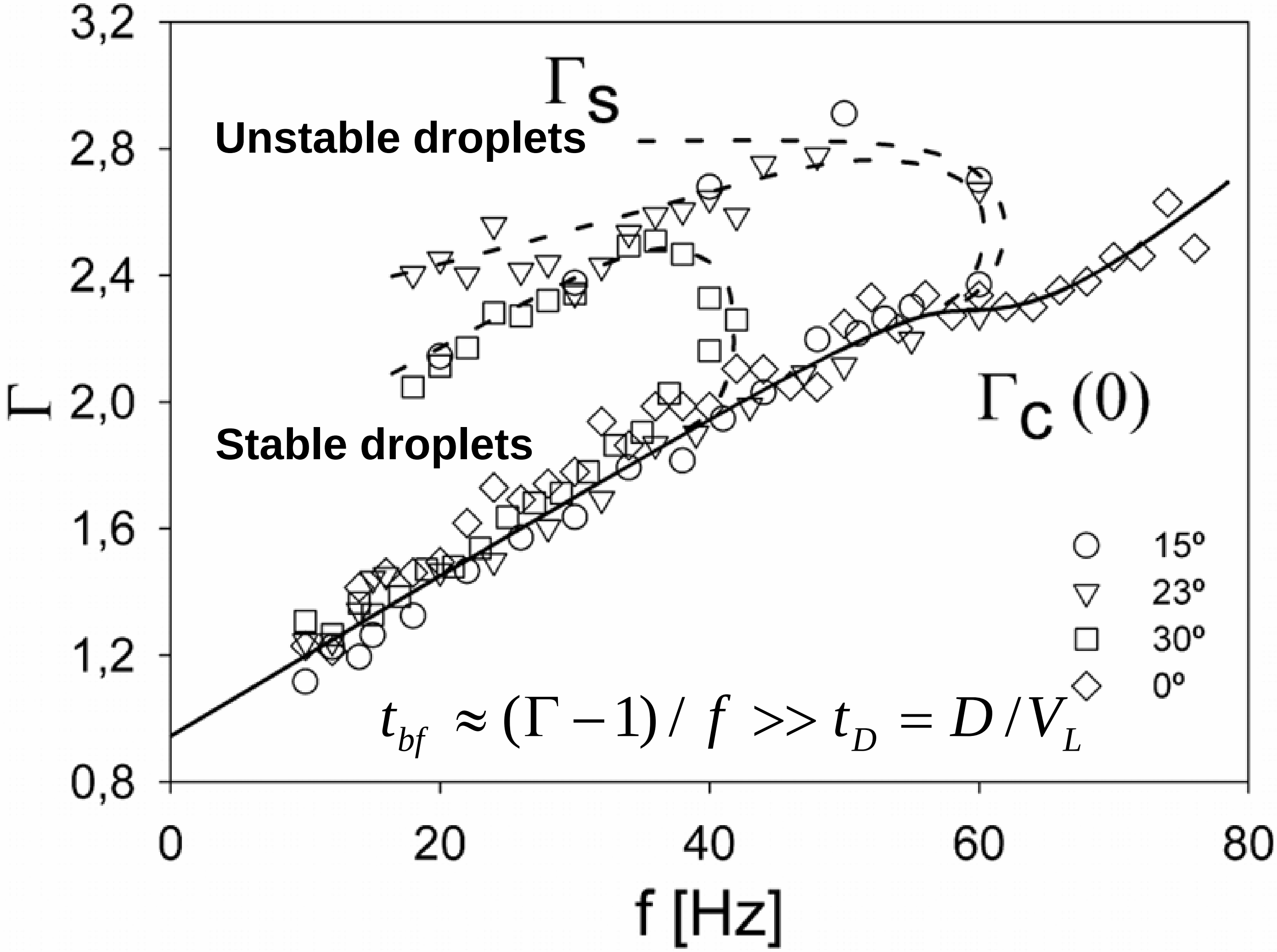
Fine powders.



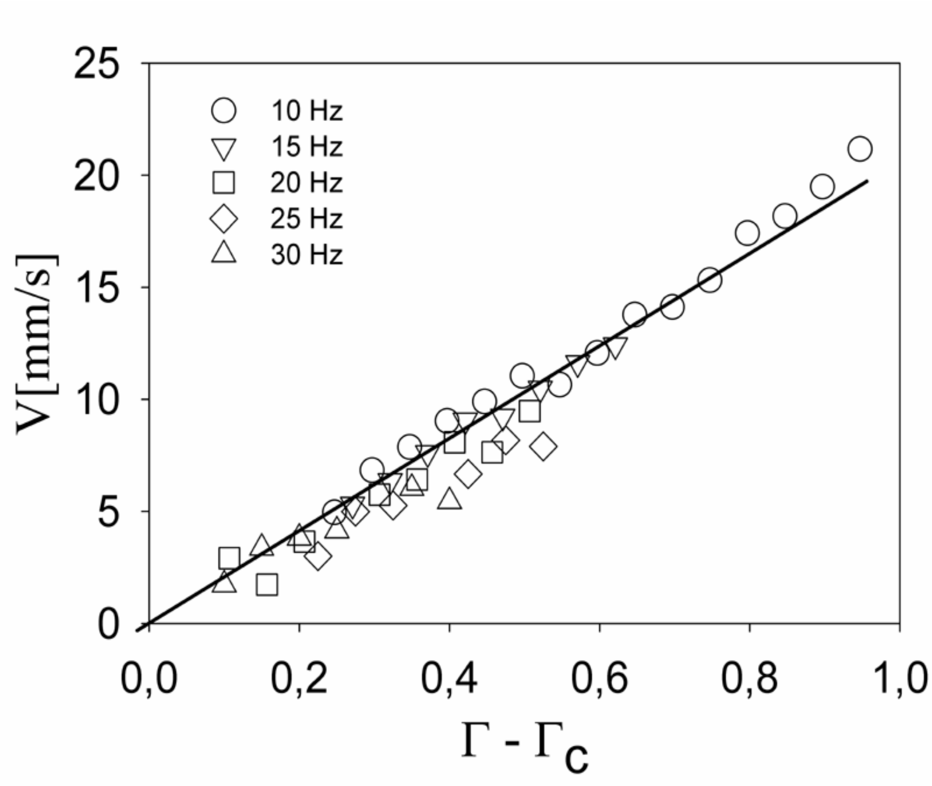


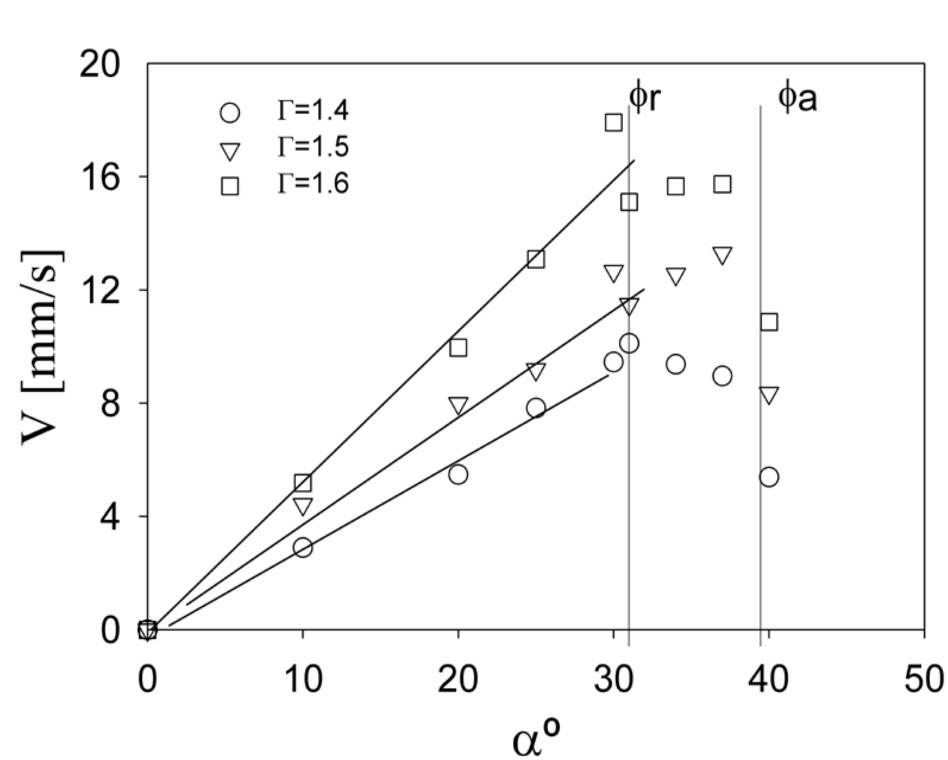
$$\Gamma = 4\pi^2 f^2 A / g$$

$$E_{inj} \propto (\Gamma - 1)^2 / f^2 = Cte$$



$$t_{bf} \approx (\Gamma - 1)/f >> t_D = D/V_L$$





$$Experiments \Rightarrow V = V_{D\eta}\alpha(\Gamma - \Gamma_c)$$

A simple picture.

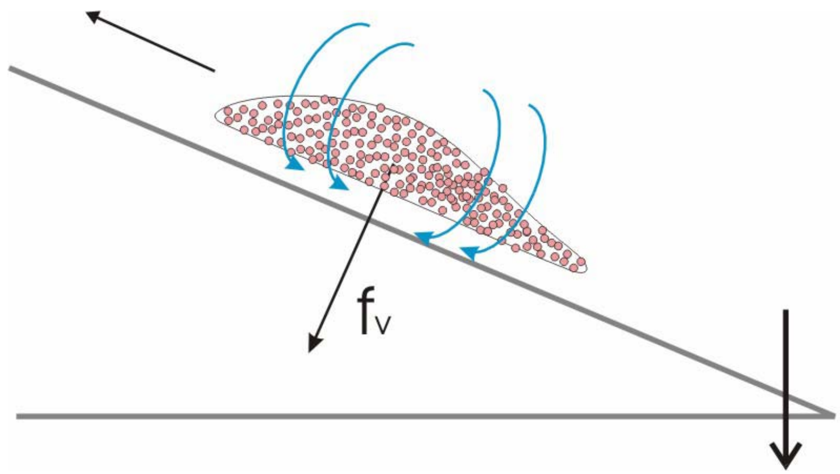
- Stokes flow: $\pi\rho D^3 g/6 \sim 3\pi D\eta V_D \Rightarrow V_D = \rho g D^2/18\eta$

Limit speed

- Darcy law: $V = \nabla P\left(D^2\phi/150(1-\phi)^2\eta\right)$

$V_L = g\rho\left(D^2\phi/150(1-\phi)^2\eta\right)$

$t_L = V_L/g$ Relaxation time



-Scaling:

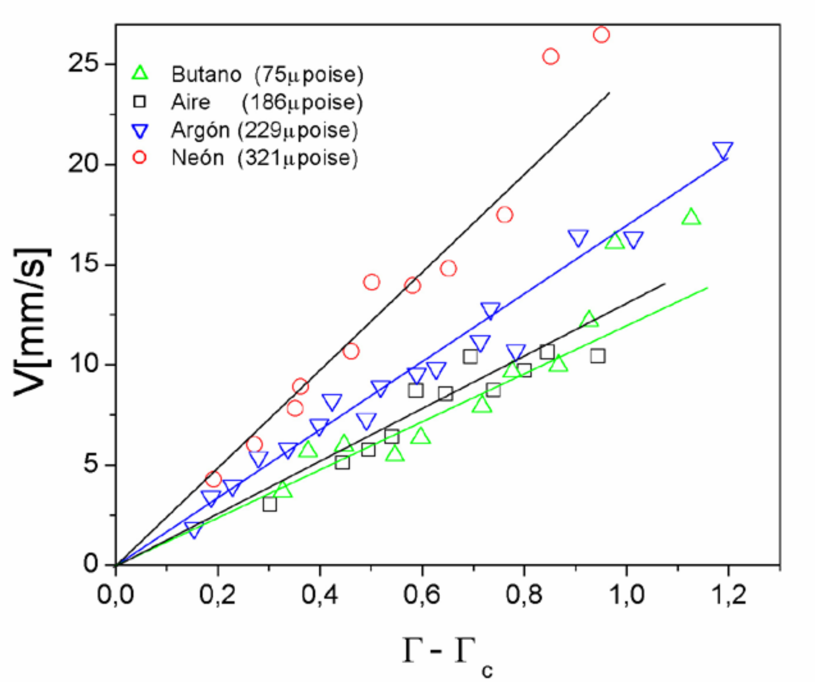
$$t_{ff} \approx \gamma(\Gamma - \Gamma_c)/f$$

$$t_D = D/V_L$$

$$g_x = g\sin(\alpha);$$

$$V = g\tan(a)t_D t_{ff} f \Rightarrow V = gD\tan(\alpha)\gamma(\Gamma-\Gamma_c)/V_L \propto \rho\eta/D$$

Fluid viscosity effect



Conclusions:

-A simple idea captures the main features of wetting droplets

-To a more elaborate description:

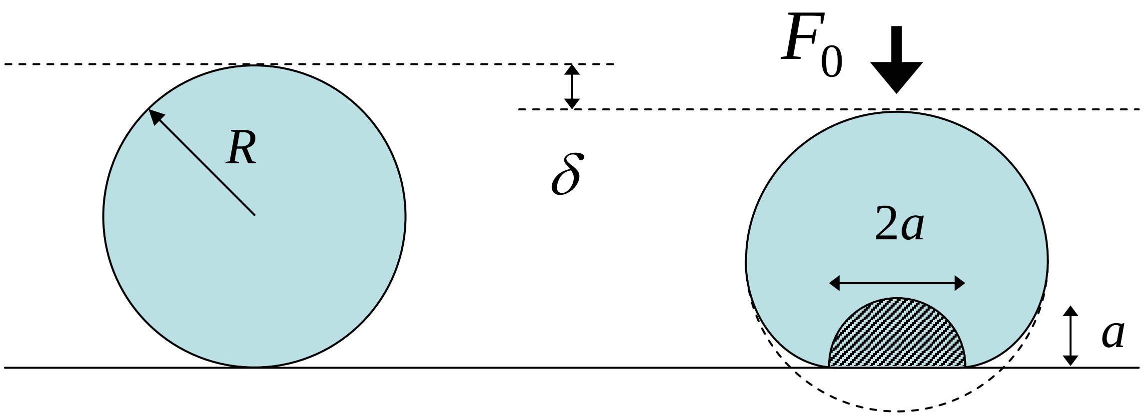
-Vary particles diameter.

-Full characterization of gas flow.

Back to Newton time!

Impulsion transmission in elastic beads

★ Nonlinear behavior of spherical contacts under elastic deformation: Hertzian contact

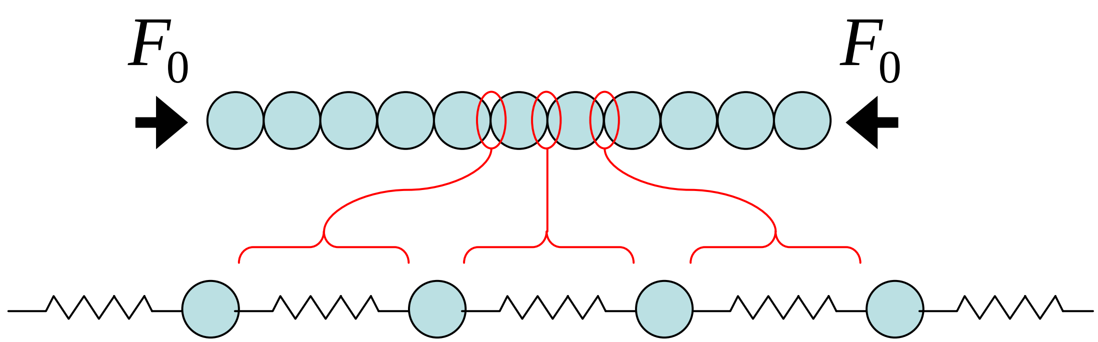


$$a^2 = R^2 - (R-\delta)^2 \approx 2R\delta \quad \longrightarrow \quad F_0 \sim ES(\delta/a) \sim E\delta a \sim E\delta^{3/2}$$

Basic concepts

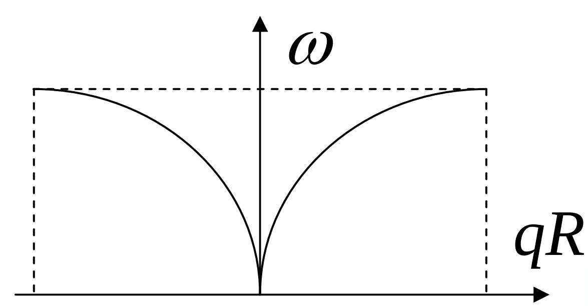
★ A chain of identical beads (mass *m*) is a dispersive medium

$$k_{eq} = \partial F_0 / \partial \delta \propto F_0^{1/3} \quad \Rightarrow \quad \Delta F_0 \approx k_{eq} \times \Delta \delta$$



$$\omega(q) = 2\sqrt{k_{nl}/m}\,|\sin(qR)|$$

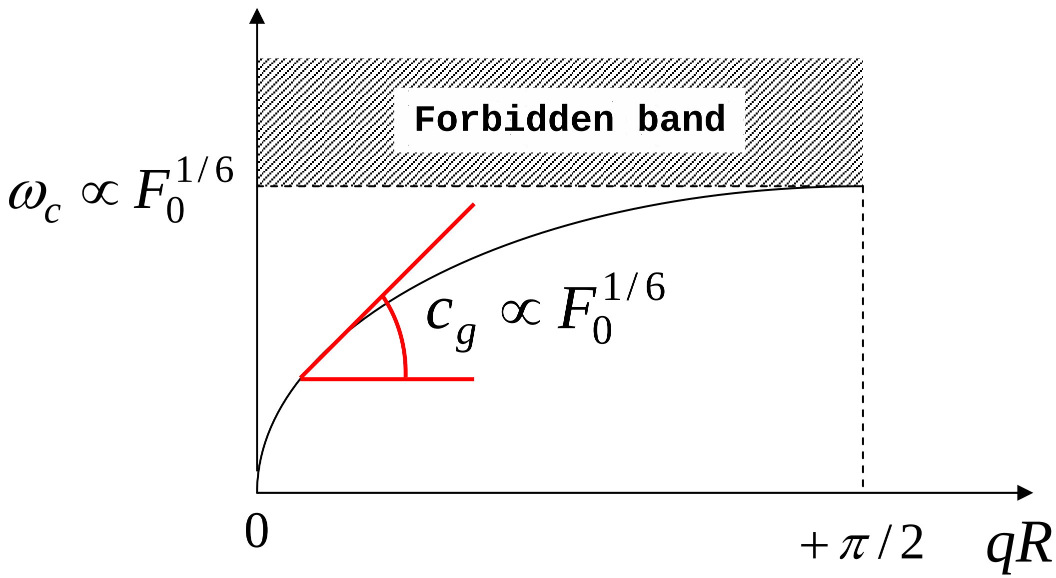
Dispersive medium: $C_\varphi \neq C_g$



Basic concepts

★ Dispersion relation for acoustical mode

$$\omega = \omega_c |\sin(qR)|$$



⇨ Acoustical modes propagate along chains of force, where $F_0 \neq 0$

⇨ $F_0 = 0$ corresponds to the *sonic vacuum* limit

Basic concepts

★ *Sonic vacuum*, but nonlinearity + dispersion = Solitons

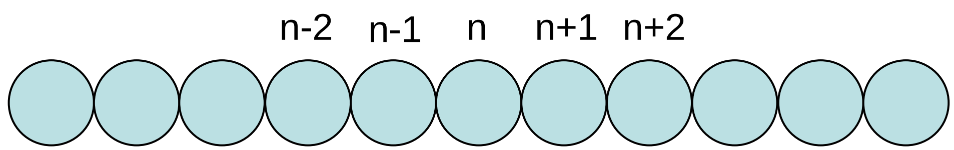
e.g. KdV equation

$$\frac{\partial \upsilon}{\partial t} + (1+\upsilon)\frac{\partial \upsilon}{\partial x} + \frac{\partial^3 \upsilon}{\partial x^3} = 0$$

Nonlinearity COMPENSATES Dispersion

⇨ A chain of elastic beads supports acoustical wave and also **solitary waves**

Modeling equation: V. Nesterenko 1984



$$m\partial_{tt}^2 u_n = \kappa\left[(u_{n-1}-u_n)^{3/2} - (u_n - u_{n+1})^{3/2}\right] \qquad \kappa = \frac{R^{1/2}}{2^{3/2}\theta} \qquad \theta = \frac{3(1-\sigma^2)}{4Y}$$

$$\lambda >> R \quad \Rightarrow \quad \partial_{tt}^2\psi + C^2\partial_{xx}^2\left[\psi^{3/2} + (2/5)R^2\psi^{1/4}\partial_{xx}^2\left(\psi^{5/4}\right)\right] \approx 0 \qquad C^2 \propto \kappa/m$$

$$\psi = -\partial_x u \qquad \psi = \left(\frac{5}{4}\right)^2\left(\frac{\upsilon}{C}\right)^4\cos^4\left(\frac{x-\upsilon t}{R\sqrt{10}}\right) \qquad F \approx \kappa(2R\psi)^{3/2}$$

$$F = F_m\cos^6\left(\frac{x-\upsilon t}{R\sqrt{10}}\right) \qquad \upsilon \approx \left(\frac{6}{5\pi\rho}\right)^{1/2}\left(\frac{F_m}{\theta^2 R^2}\right)^{1/6} \propto Y^{1/3}F_m^{1/6}$$

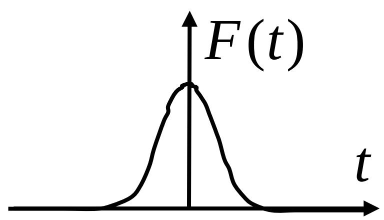
Basic concepts

★ *In summary*

⇨ $F_0 \neq 0 \longrightarrow c_g \propto F_0^{1/6}$ Acoustical modes

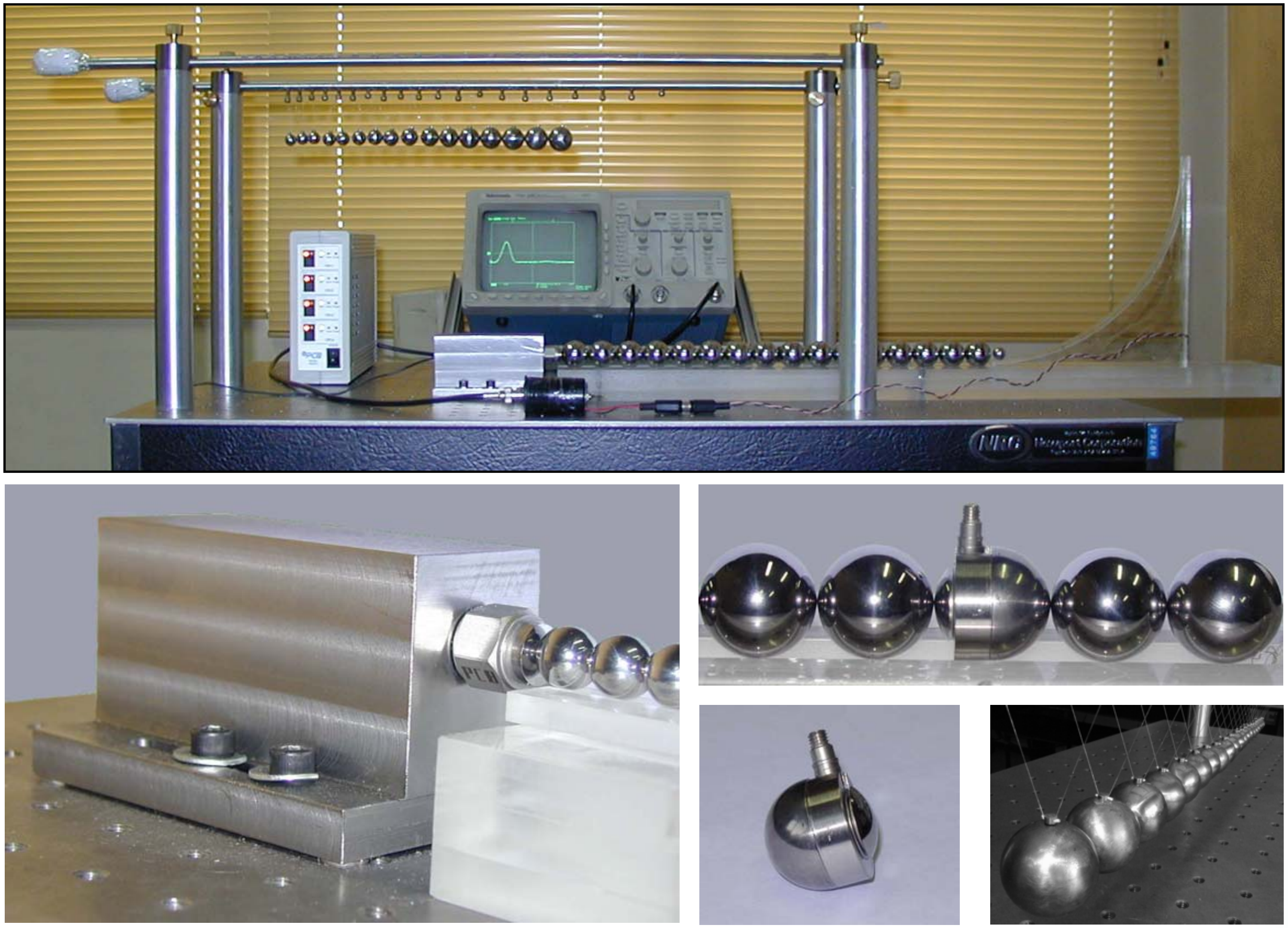
⇨ $F_0 = 0$ No acoustical wave but our system exhibits a solitary wave solution (a *mexican-hat* profile)

$$F(t) = F_m \cos^6\left(\frac{x - \upsilon t}{R\sqrt{10}}\right) \quad \text{with} \quad \upsilon \propto F_m^{1/6}$$

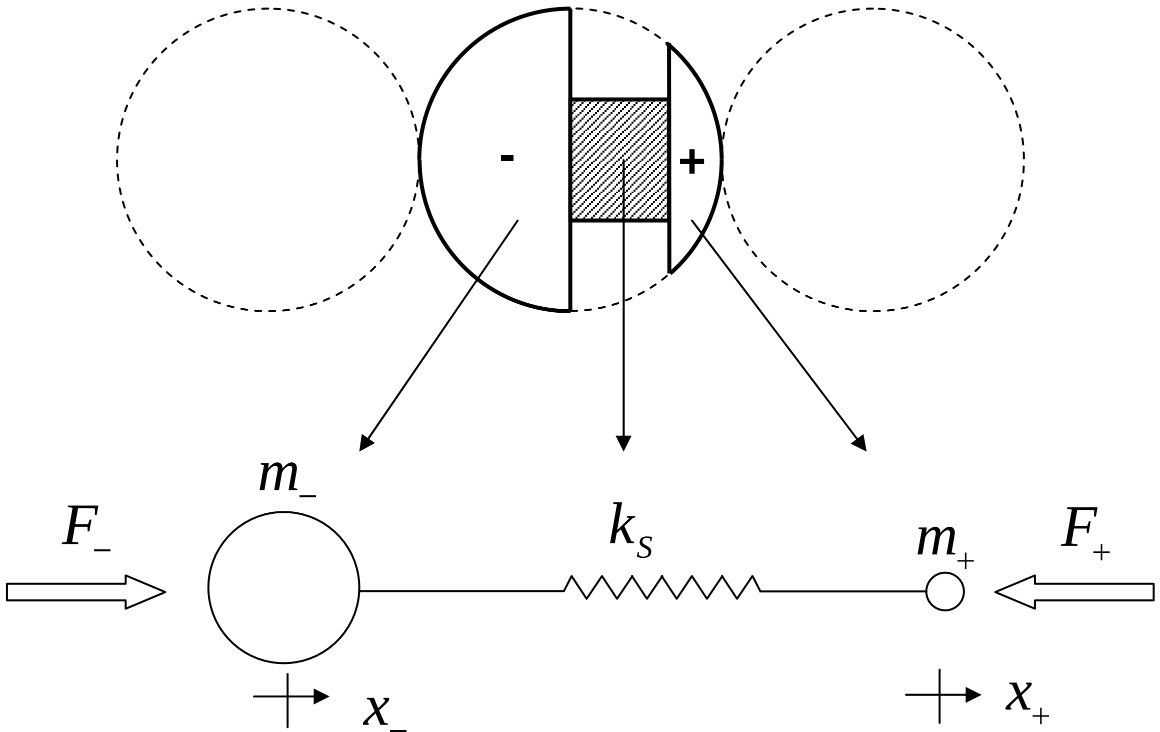


Qualitative agreement with previous experimental works. For instance, V. Nesterenko et al and E. Falcon et al.

Experimental setup



Sensor response



$$\omega_0^2 = k_S\left(m_+^{-1} - m_-^{-1}\right)$$

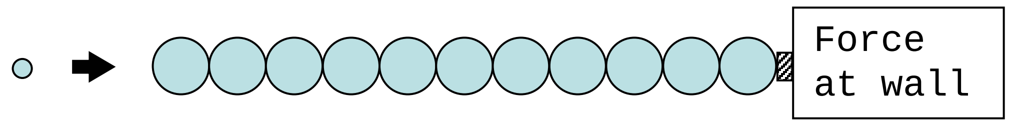
$$\varepsilon = m_+ / m_-$$

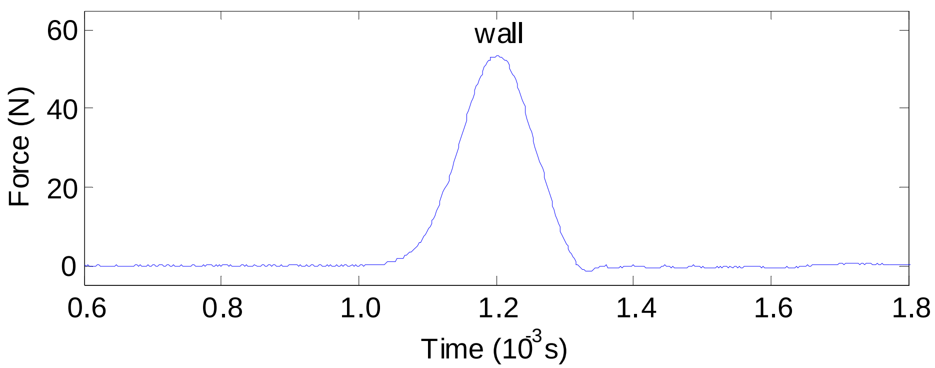
$$F_S = k_S\left(x_+ - x_-\right)$$

$$F_\pm = F_S \pm m_\pm \partial_{tt}^2 x_\pm$$

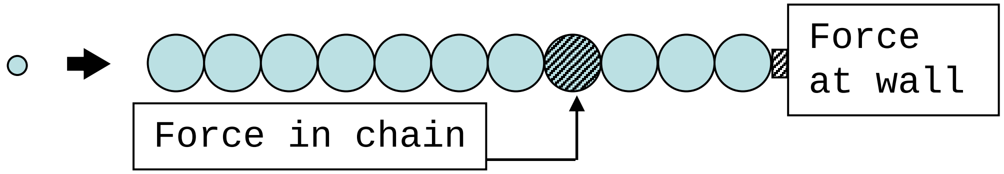
$$\Rightarrow \quad \partial_{tt}^2 F_S + \omega_0^2 F_S = \omega_0^2\left[\left(1-\varepsilon\right)F_+ + \varepsilon F_-\right]$$

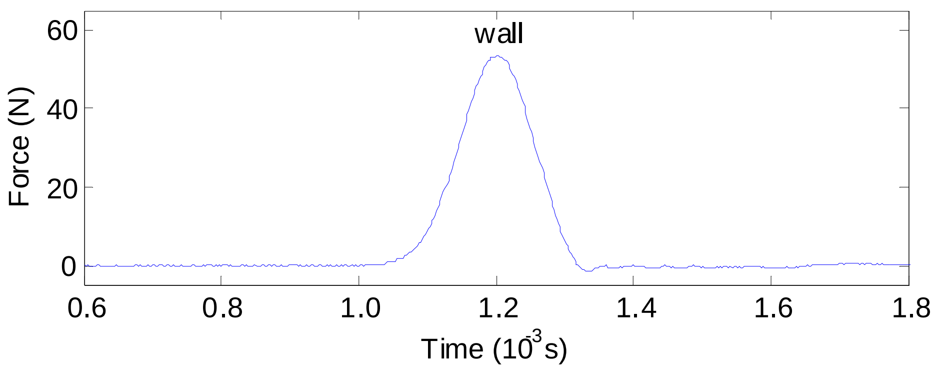
Monodisperse chain of beads



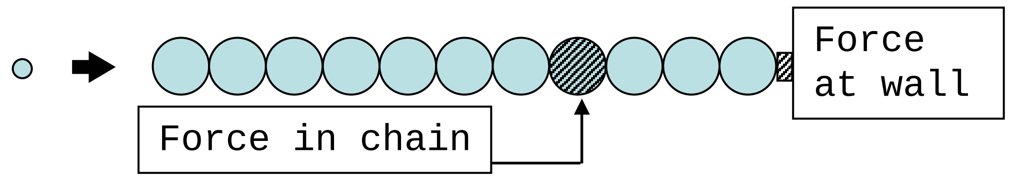


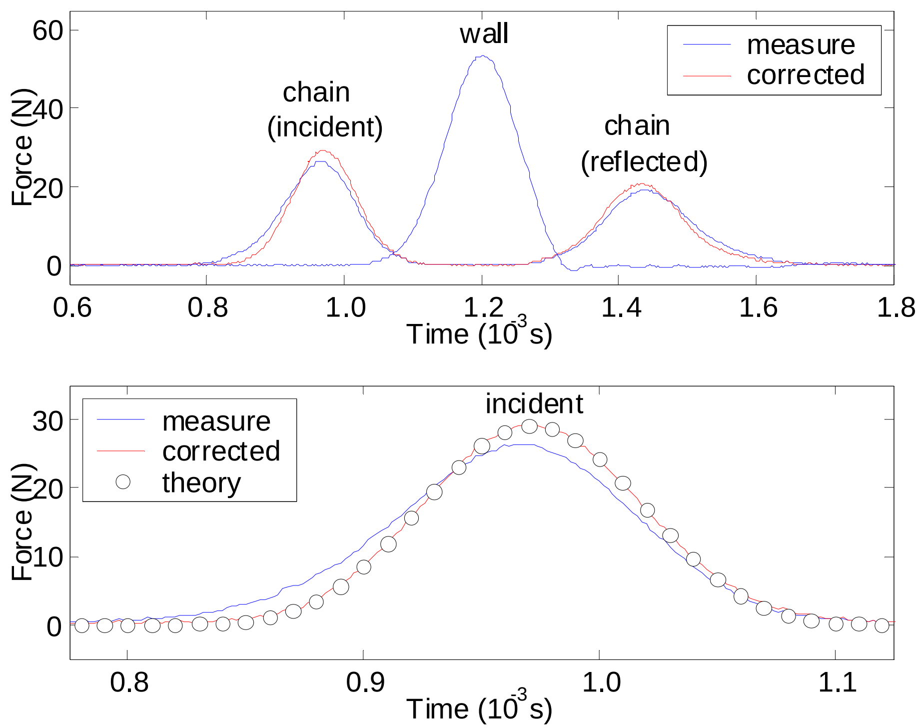
Monodisperse chain of beads





Monodisperse chain of beads



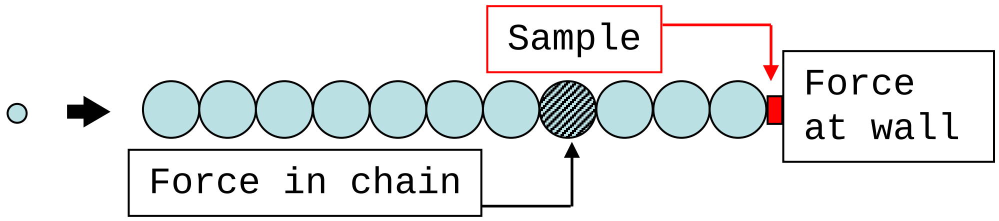


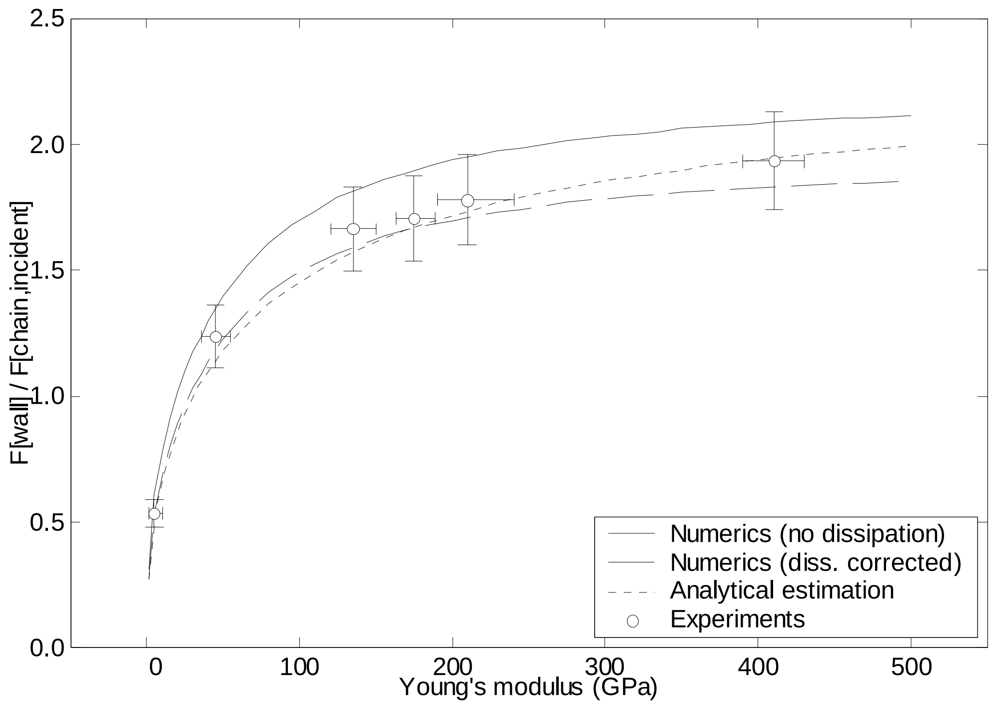
Theoretical profile

$$F(t) = F_m \cos^6\left(\frac{x - \upsilon t}{R\sqrt{10}}\right)$$

with $\upsilon \propto F_m^{1/6}$

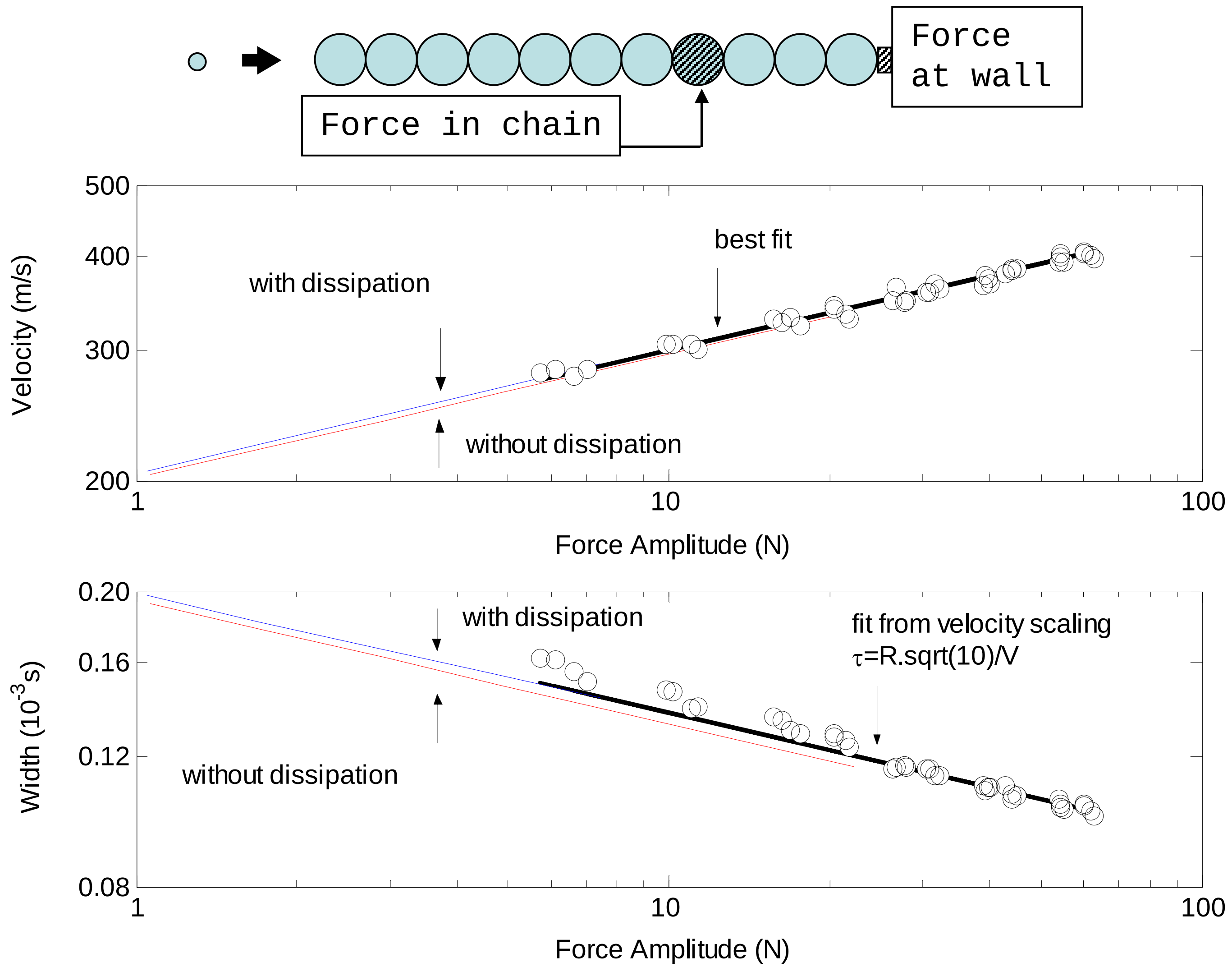
Monodisperse chain of beads





Alternative method to RUS to measure Young modulus of small samples.

Monodisperse chain of beads



Introducing dissipation

$$\psi \approx \delta/(2R) \Longrightarrow \partial_{tt}^2\delta + C^2\partial_{xx}^2\left[\delta^{3/2}\right] = 0$$

Dissipative behavior

$$\partial_{tt}^2\delta + C^2\partial_{xx}^2\left[\delta^{3/2} + \eta\partial_t\left(\delta^{3/2}\right)\right] = 0$$

$$\partial_{tt}^2\delta + C^2\partial_{xx}^2\left[\delta^{3/2}\left(1 + \frac{3\eta}{2}\frac{\partial_t\delta}{\delta}\right)\right] = 0$$

$$\partial_t\delta \approx \pm\delta_m/\tau \Longrightarrow \partial_{tt}^2\delta + C^2\partial_{xx}^2\left[\delta^{3/2}\left(1 \pm \frac{3\eta}{2\tau}\right)\right] = 0$$

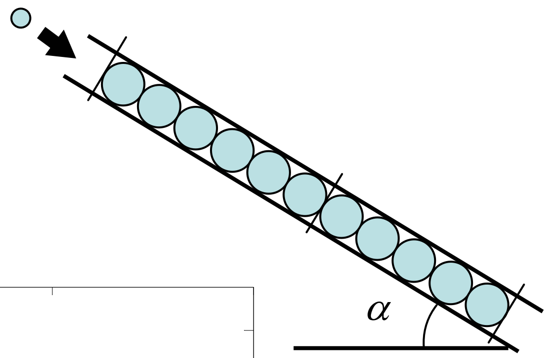
$$C^2 \propto Y \Longrightarrow Y^* = Y\left(1 \pm \frac{3\eta}{2\tau}\right)$$

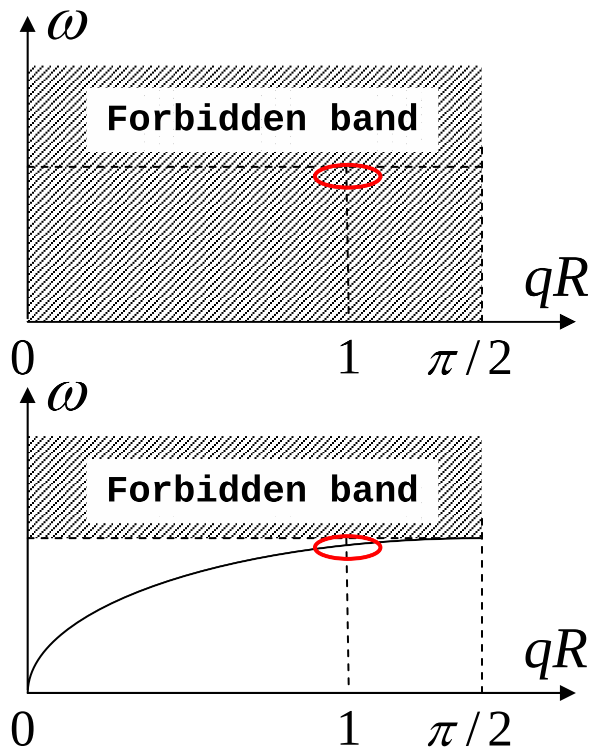
→ Compression (release) feels harder (softer) medium when dissipative

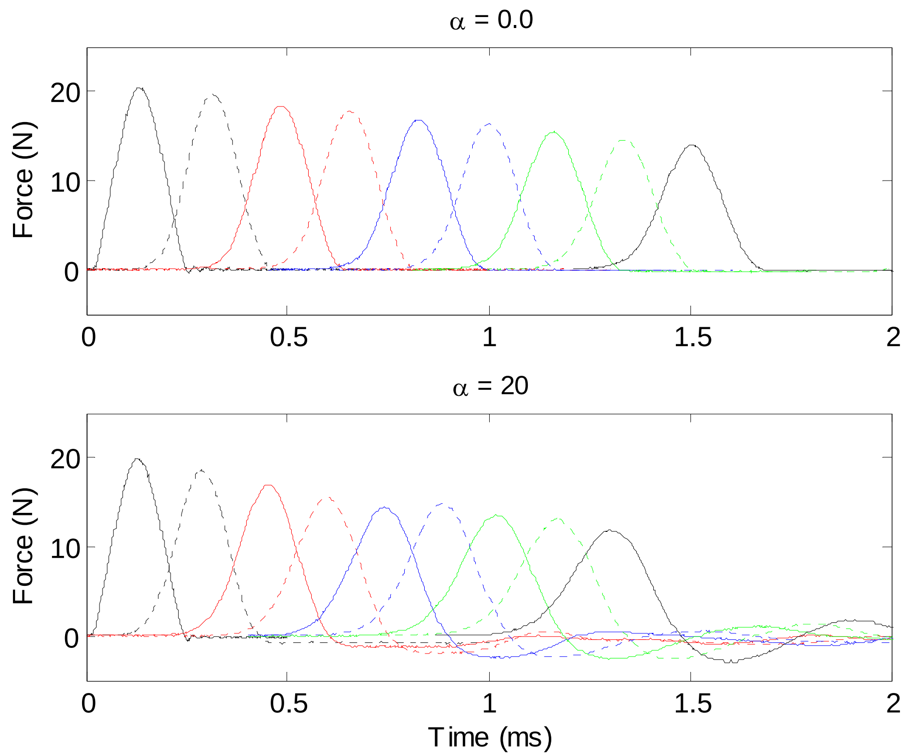
→ Compression faster than release i.e. broadening of the soliton

Effect of the gravity

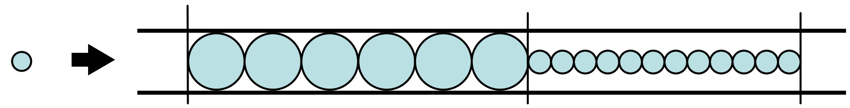
★ Gravity introduces a gradient of static force

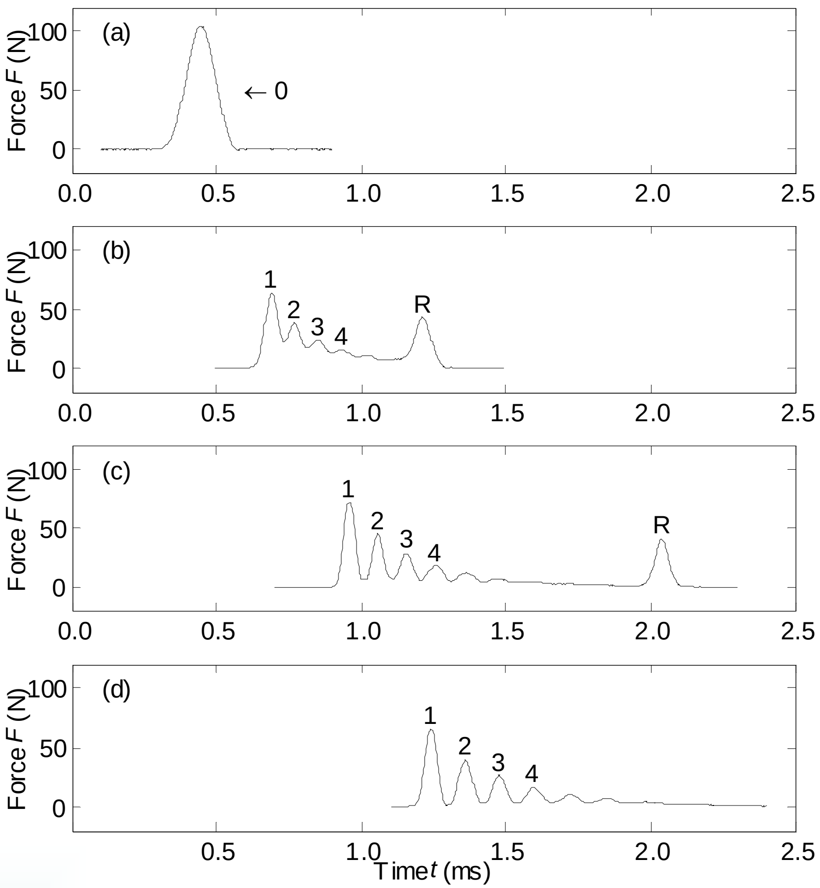






Impedance mismatch





Almost all the energy is transmitted.

perspectives

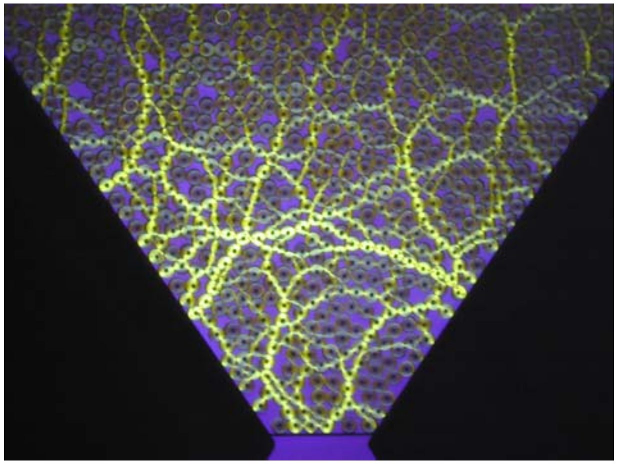
-Good agreement between experiments and theory under academics configurations. Need a more accurate theory, including dissipation

-Study of a disordered chains, and few others configurations.

-Two and three dimension effects: disorder, shock front?

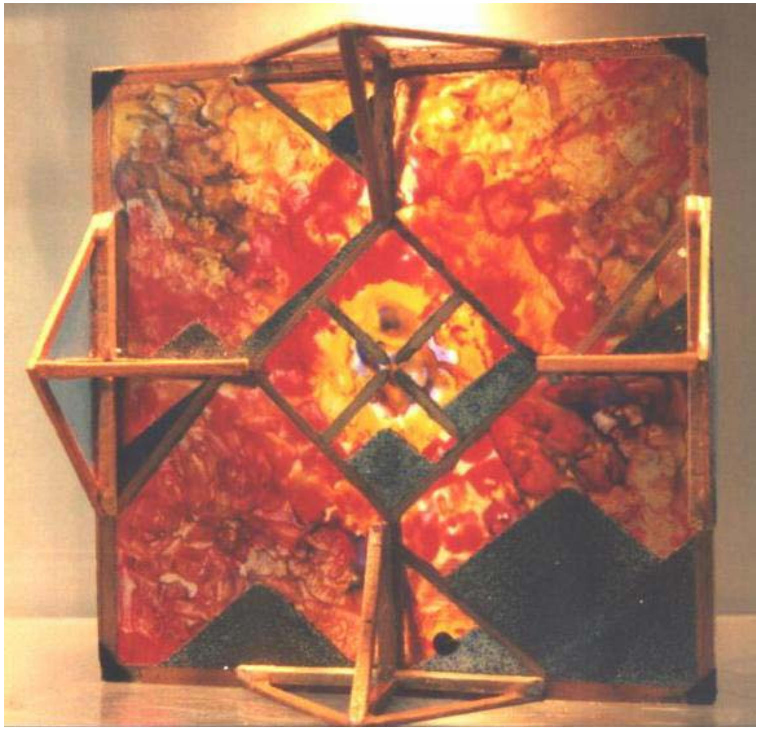
- Shape effects, Hertz is no longer valid but solitary waves might remain; introduce defect contact to check.

In 3D, acoustical modes propagate along chains of force, where $F_0 \neq 0$



$F_0 = 0$, regions of solitary wave propagation.

Conclusions.



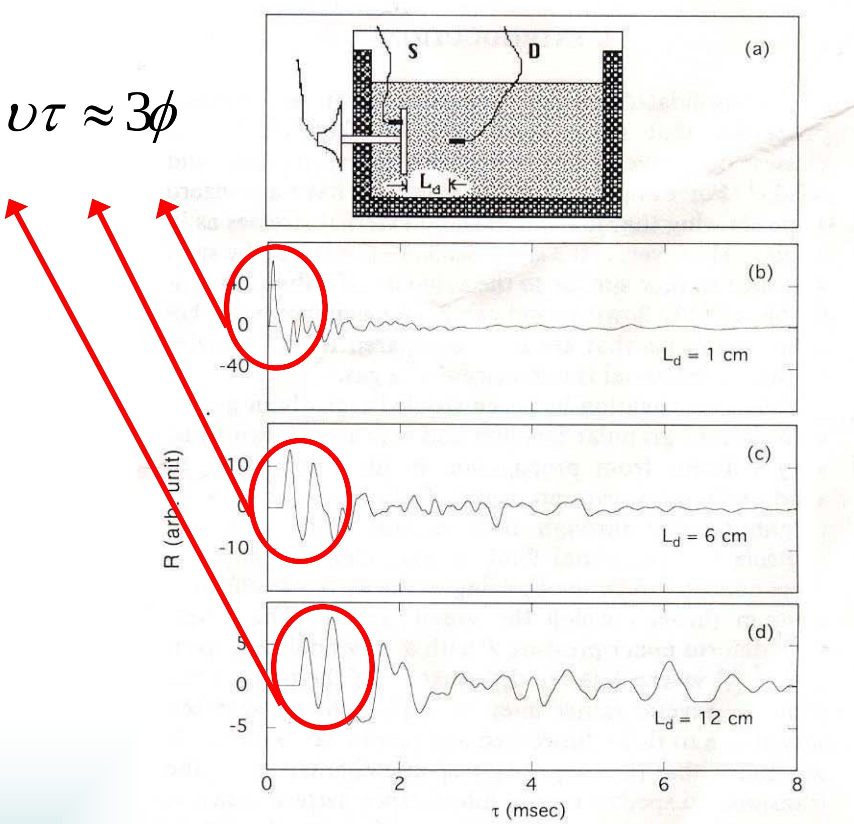
Works of 10 years old kids

Updated version of the manual of granular material exhibit.

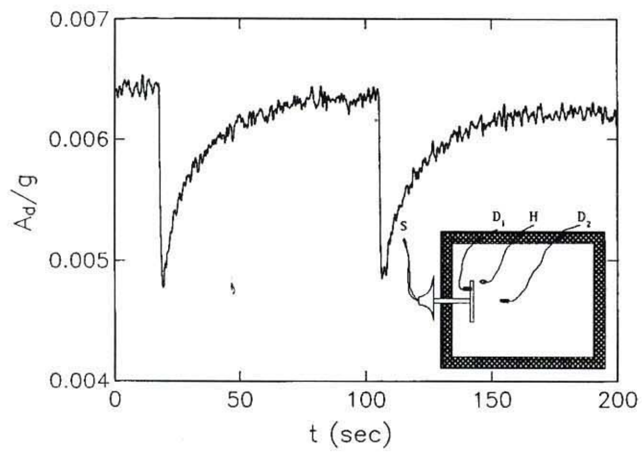
Extension to 3D medium

★ *"Sound in sand"*, Liu, Nagel, PRL 68(15), April 1992.
"Spatial patterns of sound propagation in sand",
Liu, PRB 50(2), July 1994.

$\upsilon\tau \approx 3\phi$



→ Effect of 1D chain of force

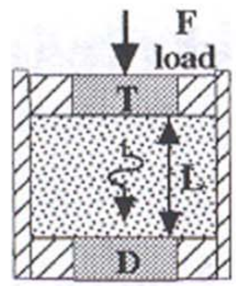


Extension to 3D medium

★ *“Ultrasound propagation in externally stressed granular media”*, Jia, Caroli, Velicky, PRL 82(9), March 1999.

Coherent ballistic signal

Multiply scattered sound



The coherent part of the signal follows Hertz law

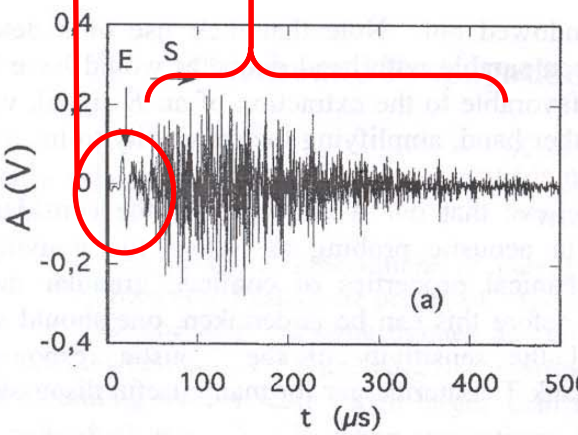


FIG. 3. Ultrasonic signals through the same bead packing as Fig. 1b detected by a smaller transducer under the same normal stress $P = 0.75$ MPa: (a) First loading; (b) reloading.

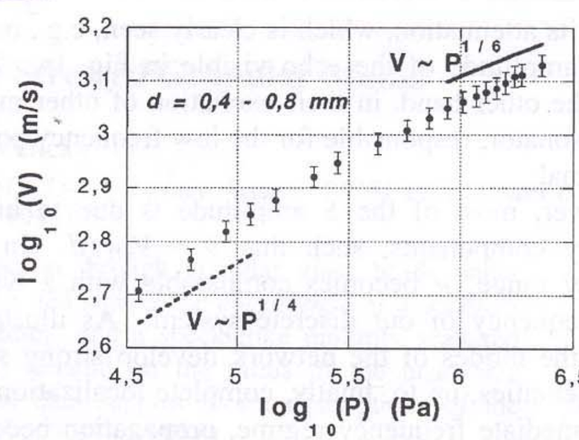


FIG. 4. Sound velocity V (data points) of the coherent E wave in the bead packing, $d = 0.4-0.8$ mm versus the applied stress P.