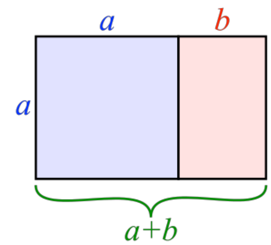
THE UNIVERSITY OF CHICAGO

The 77th Arthur H. Compton lecture series
The Enrico Fermi Institute
The James Franck Institute

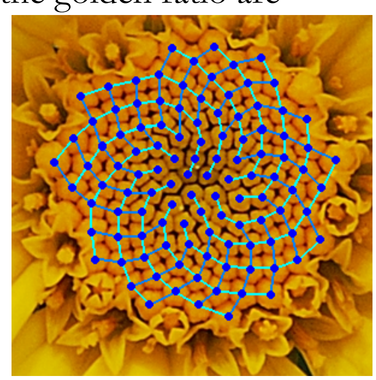
Lecture 8 June 1st 2013

Phyllotaxis, The golden ratio and the Fibonacci sequence



The golden ratio is a number, φ, such that the ratio between φ and 1 is the same as the ratio between 1 and 1- φ. It is an irrational number, given explicitly by $\frac{1+\sqrt{5}}{2} = 1.618033988\ldots$. It was first described by Euclid, and can be visualized as the aspect ratio of a rectangle composed of a square and a smaller rectangle also having the aspect ratio φ. It has unique properties and arises in nature in surprising circumstances. However, in popular culture there are also a lot of myths associated with it and many false identifications of the golden ratio.

The Fibonacci sequence is a number sequence starting with two ones, then iteratively constructing the next number as the sum of the previous two. The first few numbers of the sequence read: (1,1,2,3,5,8,13,21,34,55,89,144…). An interesting (and easily provable) fact is that the ratio of consecutive Fibonacci numbers converges to the golden ratio. Conversely, good rational approximations of the golden ratio are comprised from numbers from the Fibonacci sequence.



Phyllotaxis, the arrangement of leaves on a stem, and the arrangement of seeds in some flowers, often display regular spiraling patterns. When counting the numbers of spirals (in either of the directions) one often (but not always) finds a familiar set of numbers like 13 and 21 displayed here, and also 5,8,34,55 and 89. The reason these plants display numbers from the Fibonacci sequence is closely tied to the properties of the golden ratio. One can show that the golden ratio is not only irrational but has the “worst possible” good rational approximation amongst all irrational numbers. This property lent the golden ratio the title “the most irrational number”. It makes the arrangement of successive leaves separated by the golden angle optimally distributed, and can therefore be also realized by inanimate physical systems.

The golden ratio also appears naturally in the length of the diagonal of a pentagon, and thus in systems displaying pentagonal symmetry. The Penrose tiling is an example of a system whose underlying building blocks were derived from a pentagonal structure and display the golden ratio. Despite its many qualities and surprising emergence in natural phenomena people tend to also associate the golden ratio with systems it has very little to do with, from the spiraling shapes of the nautilus shell and of our galaxy, to the aspect ratio of the Parthenon.