Part 5 Homework

v.1 Are there additional solutions to equation v.12, beyond the one we gave? Discuss this issue.

v.2 Assume our Brownian particle, as described by equation v.13a, is charged? How can one include electric fields in this analysis? Magnetic fields? Does the system go to equilibrium in the presence of space and time-independent fields? What happens when the field depends upon time?

v.3 How can we be sure that equation v.13a conserves the total probability of finding the Brownian particle? Should it conserve the momentum or energy of that particle? What are the equations for the time dependence of the particle’s energy and momentum? What about its angular momentum?

v.4 Find the local conservation laws for energy and momentum from the Boltzmann equation, equation v.14.

v.5 Draw a series of pictures or construct a little video to show the motion of a region of phase space determined by the motion of a

v.6 Edward Lorenz saw chaotic solutions to the equations

\[
\begin{align*}
\frac{dx}{dt} &= 3 (y-x) \\
\frac{dy}{dt} &= 26.5 x - y - xz \\
\frac{dz}{dt} &= xy - z
\end{align*}
\]

What happens to volumes in the (x,y,z) space in the time development of these equations? Notice that these equations have a time-independent solution x=y=z=0. What is the motion near this point?

v.7 Calculate the probability that a one-dimensional discrete-lattice random walk will return to its starting point after M steps, for M=1,2, and three and for large values of M. Do the same for two and four dimensions.