Singular Shapes in Two Dimensions

For almost two decades, the DMR support of Leo Kadanoff has been devoted to the analysis of mathematical singularities in the shape of objects in the plane. These cusps, points, and fractal shapes have been seen in the shapes of interfaces between different liquids and in interfaces in phase transitions. In the last four years the subject has exploded as mathematicians have invented a new technique, SLE, for generating shapes. The shape appears as a singularity in a differential equation whose time dependence is governed by a driving function.

DMR grantees I. Gruzberg and L. Kadanoff have written an exposition for the Journal of Statistical Physics aimed at bringing this new subject to the attention of a wider audience. Kadanoff, working in the Netherlands with Kager and Nienhuis, has found new solutions to the basic mathematical equations of the theory and has a new and quite precise result for when the system does form singularities. This work is being continued by Kadanoff and Marko Kleine Berkenbusch, a DMR0094569-supported student who is looking at the structure of the generated singularities.

SLE can be thought of as a machine which generates a two dimensional shape starting from an input which is a single function of one variable. The result has beautiful properties of scaling and universality. The figure shows a the repeated scaling of the shape generated by the input $t \times \sin (\pi t)$. 

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