I. INTRODUCTION

When studying a low-energy, and hence low-frequency, phenomenon through some form of electromagnetic radiation, one possible feature of the environment that can corrupt data as well as saturate the instrumentation is high-frequency radiation. In order to avoid this, filters that transmit low-frequency radiation but block high-frequency radiation can be used. One method of blocking high-frequency power is reflection, but this may cause the radiation to remain inside the instrument since it is not eliminated. A second way to block is attenuation by absorption or scattering, converting the energy to internal atomic or molecular motion or causing most of it to be lost through multiple scatters among particles. For my investigation, I tested different materials in order to find a better low-pass filter than is currently in use for the range of 5-30 cm$^{-1}$ (150-900 GHz). This range of frequencies, which encompasses the low-energy end of the far-infrared as well as high-energy microwaves, is worthwhile to explore, as the Cosmic Microwave Background (CMB) peaks within it, at roughly 6 cm$^{-1}$. Particularly important for the studies of the CMB involving bolometers, light-sensitive devices that will record a voltage as a function of incident intensity, is the removal of the high-frequency radiation sources in the sky that are not the CMB itself.

A form of far-infrared low-pass filter in bolometer-based CMB experiments is 25% glass-filled polytetrafluoroethylene (PTFE, or Teflon), sold under the name of "Fluorogold" as a low-friction bearing material [1]. The glass fibers within Fluorogold absorb and scatter incoming radiation in a frequency-dependent manner. High-frequency light is more likely to be absorbed or scattered because a higher frequency implies a smaller wavelength, and the smaller the wavelength of the light, the closer it is to the size of the microscopic glass fibers and their constituents. To improve on Fluorogold, I had a special production of Teflon made with a 50%-filling of glass spheres. I hypothesized that such material could be a more effective filter for three reasons. First, spheres should be better than fibers in a filter because a fiber, being long in one dimension but much thinner in the other two, is an efficient absorber/scatterer in only one cross-section, but I decided upon a one-to-one ratio between glass and Teflon because approaching the limit of having all glass (logical extension of more glass being better) is not more desirable, since individual pieces of glass provide discrete physical objects off which radiation is able to scatter. This phenomenon of scattering off inhomogeneities is Mie scattering [2]. Third, a fiber-shaped object radiates in direct proportion to frequency, when on the other hand a sphere radiates as frequency squared [3]. Therefore, it is more probable for spheres to absorb and re-radiate incoming radiation at higher frequencies.

II. PROCEDURE

In addition to glass-filled Teflon, I also tested pure Teflon, as well as other polymers, both glass-filled and not; namely, black high-strength, high-density polyethylene and black 25% glass-filled polycarbonate. I considered the effects of four different conditions: sample manufacturer, temperature, and, to a lesser extent, age and geometry. I further subdivided geometry into three categories: orientation with respect to the incident light, shape (cylinder versus sheet) and thickness, but had time to only consider the latter two.

The manufacturer is a significant factor.
because different manufacturers may produce different forms of glass-filled Teflon. Since it is an inhomogeneous mixture, its method of production is not standard. Even samples from vendors all claiming the same percentage of glass may differ in terms of spectral properties, depending upon how they merged the glass with the dielectric substrate during the production phase. Age can be a factor too because Teflon is a polymer which can change its properties with exposure to UV and ozone [4,5]. On the scale of decades, they will flow. It may therefore be possible for the glass within a particular sample to change location and orientation over time. In addition, vendors may revise their formulation or method of creating the mixture over time without notifying consumers. Lastly, temperature affects the optical properties by changing particle motions or the distribution of atoms and molecules, in terms of energy levels. I studied most of my samples at room temperature (~300 K), but I also studied the most promising low-pass filters candidates at liquid nitrogen temperature (77 K) and liquid helium (4.2 K). The purpose of cooling to 4.2 K was to reduce the intrinsic radiation of the test sample. It is desirable to minimize unwanted radiation, and thermal emission from a 300 K body is substantial. A reason for exploring the effect of 77 K as well was to characterize the change in the behavior of a substance as a result of changing temperature, in a quantitative fashion in its spectrum.

The experimental setup was as follows:
light from a blackbody of a known temperature (1020 K) entered a Fourier Transform Spectrometer (FTS), the explanation of whose function and operation is in the next paragraph. After leaving the FTS, the radiation passed through metallic light pipes (copper, aluminum, or stainless steel), being guided through a sample and then through a turning point, which was a U-shaped piece of copper tubing. Radiation able to pass through the sample traveled along another light pipe, at the end of which it encountered a bolometer, which measured the intensity of incident electromagnetic radiation via a voltage as a known function of absorbed power. A computer attached to the bolometer read the voltage across it.

The FTS, a polarizing Michelson-Morley interferometer, provides a powerful technique for determining the transmission fraction through a sample. Spherical wavefronts from a blackbody source at the focus hit a converging mirror and become plane waves. Radiation next encounters an input polarizer, which reflects the horizontally polarized component but transmits the vertically polarized one. Each part will have half of the original intensity. The reflected portion hits a filter at a 45° angle known as the beam splitter. The beam splitter will transmit light with a 45° angle of polarization but reject (that is, reflect) light at 135°. However, the reflected light then encounters a dihedral mirror (Figure 2), which alters its polarization to 45°, so that this light now passes through the beam splitter. The light originally able to pass through the beamsplitter, bounces off the other dihedral mirror, which converts its polarization to 135°, so that it is now no longer transmitted, but instead reflected from the beam splitter. The end result is a full recombination of the two split beams. The vertically polarized half of the radiation input to the FTS experiences an analogous process. By means of another mirror, the FTS forces the vertical and horizontal components to both reencounter the initial polarizer, which will transmit the vertical but reflect the horizontal. In this way, they recombine at an output port since they come at the input polarizer from opposite sides. Therefore, none of the incoming radiation is lost. A computer controls the phase delay between the beams accepted and rejected at the beam splitter by systematically varying the location of a carriage carrying the dihedrals.

A path difference generates the phase lag.

Taking the Fourier transform of the output intensity as a function of optical path difference, that is, the Fourier transform of the interferogram, provides one with the output intensity as a function of frequency, that is, the spectrum. I used a spectrum taken with no sample in the optical path as a reference. That means that in order to find the transmission spectrum of a given sample, I took the Fourier transform of its interferogram and divided it by the null spectrum. The result: a transmission coefficient (0-1). The reason a Fourier Transform is able to take me from intensity as a function of path difference to a function of frequency is the fact that phase and frequency are conjugate variables, in the sense that phase lag in real space translates into frequency in Fourier space. For more information, see Chamberlain [6].

![Diagram](image)

LEFT: Figure 2. Fourier Transform Spectrometer.
Obtaining transmission measurements at cryogenic temperatures required a cryogenic filter holder and filter wheel. The wheel had four apertures, three for filter material candidates, one for a blank spectrum. The wheel was constructed of Oxygen-Free Extreme-Temperature Copper (McMaster-Carr Alloy 101). After first drawing a diagram to-scale on a computer in order to make a careful study of all the relevant dimensions first, since the final product had to fit into a 10 cm-diameter glass dewar for low-temperature measurements. Once I had a circular piece of copper in hand, my next step was to create holes for the samples. I decided upon three-stage holes, that is, large holes with smaller-width holes at the same depth within them, so that the wheel would hold the test samples more stably. I decided upon only four apertures because that was the maximum allowed given the planned diameters of the wheel and apertures compounded with the demands of symmetry and the constraint of allowing enough space between the apertures to prevent more than one test sample from interacting with incoming light at once. Because of unavoidable imperfections in the stainless steel and copper light guides, the sample-less spectrum was not simply a flat line close to 100% transmission.

The next step was the creation of some mechanism so that I could attach the wheel to a pre-existing steel shaft, bearing, and knob system, designed to allow me to switch between test samples blindly, while the entire cryogenic filter holder is still inside the test dewar. The purpose of this was so that nothing hindered me from making spectra close together in time, disturbing the experimental setup least. Since one must always make relative measurements, as I described above, this was key. Disturbances in equipment may make slight changes in the reference spectrum. Using a small, hollowed-out brass cylinder with two through-holes for 5/32 set screws as the bottom of the copper filter wheel, I affixed the wheel to the shaft seen in Figures 3a and 3b. That steel shaft, which I cannibalized from existing apparatus from past, similar experiments, had a copper piece lying along it harboring a bearing that permitted the rotation of the whole system from a screwed-on knob at top. The setup ensured that I was capable of deciding on which sample to test from the outside, with the filter-holding device featured in Figure 3a in my dewar.

Next, I built a system for knowing when an aperture is properly aligned along the optical axis defined by the tubing. The idea that I ended up implementing was the following. I carefully machined a small piece of spring tempered steel. Then, I placed clearance holes for screws. I planned on affixing a ball bearing to this strip of spring steel for the purpose of riding along the copper wheel, with two screws holding the steel strip to the aluminum piece that holds the pair of stainless steel tubes together (Figure 3a). To that end, with a 1/8 inch diameter steel ball in mind, I placed a tiny hole (one-fourth of that diameter) in the spring steel. Using a tapered center punch, I ensured that this aperture would be sloped, making it easier to glue the ball into it. With Styrofoam 2850 FT Black Epoxy and a room temperature catalyst, Catalyst 24LV, I glued the steel ball bearing onto the spring steel. Differential contraction at cryogenic temperatures fortunately did not destroy that ball-strip system.

With the spring steel and ball bearing ready and screwed into the aluminum, I next etched 45° triangular notches into the wheel. Then, after reattaching the wheel to the shaft, I examined how the ball interacted with those notches. I heard an audible click whenever the wheel became properly aligned such that one of the holes was correctly centered along the optical axis, as desired. Not relying upon gravity alone to hold filters in their respective holes, however,

**Comment:** Here again, it is important that the wheel had a detents that allowed you to set the angle of the wheel with high repeatability and precision.

**Comment:** I would not give the blow by blow of the construction. It is not critical for the experiment that you drew the filter wheel with a CAD system. What is critical is that the filters were such and such a diameter, filled the aperture... It is also important that to state here what the solid angle of illumination and other properties important for the operation of the experiment. State here that the filters are immersed or are in contact with cold He gas while under test. How did you know that the filters were cold?

**Comment:** State here only that the wheel was on a bearing that worked cold and was controlled by a knob in the warm.

![Diagram](image)

**ABOVE:** Figure 3b. Clamping of copper wheel system. Notches provide physical resistance and a clicking sound that should both signal proper centering.
I developed a clamping mechanism in order to secure them more tightly in place. Using a high-speed drill, I cut four more strips of spring-tempered steel shim, which I attached to the copper by means of 2-56 screws through their centers. In order to prevent these strips from blocking or reflecting any incoming radiation, I ground them down to better shapes and sizes with a high-powered grinding wheel. Every sample thus experienced the force of two strips. However, in the case of thick material unable to fit under the strips, I used either a 5-minute epoxy or Glue Varnish in order to attach them to my wheel. From sheets and rods, I fashioned my low-pass filter candidates. Light went through these samples and hit two 45° copper plates in succession to redirect it outward, back out of the primary apparatus and into a bolometer for analysis of the amount of transmitted intensity as a function of frequency (Figure 4). I chose copper for the U-turn light guide and the filter wheel too since its good conduction of heat ensured thermal equilibrium with any liquid cryogen I used (helium and nitrogen). The stainless steel tubes were present in order to sustain a vertical thermal gradient that prevented rapid boil-off of liquid helium or nitrogen. A different metal would conduct too much heat from the outside environment down into the dewar, but I had to choose some type of metal to ensure a high infrared reflection coefficient, so radiation was not attenuated too much.

III. TRANSMISSION SPECTRUM MODEL

The transmission spectrum of a flat slab of dielectric can be written as follows. This closely follows the derivation in Halpern et al. [7]

A material with no optical losses will have a purely real index of refraction "n." But let us generalize this to a complex refractive index "n-hat:"

\[ n = \sqrt{\varepsilon \mu} \Rightarrow \hat{n} = \sqrt{\frac{\varepsilon}{\mu}} \] (1)

Arrows I shall use from now on in order to designate the generalization from non-lossy to lossy, from left to right. Hats signify the complex version of a variable that has a real-only counterpart in the non-lossy case with the same single-letter name. Imaginary parts therefore are always keeping track of the lossiness.

The dielectric constant, \( \varepsilon \), and the magnetic permeability, \( \mu \) are fit parameters for the model of the material.

For a non-lossy slab of thickness \( d \) and index \( n \), the optical admittance as a function of frequency \( v \) is given by the expression

\[ Y(v) = \frac{n \cos(2\pi v d)/n \sin(2\pi v d)}{n \cos(2\pi v d) + i \sin(2\pi v d)} \] (2a)

where \( v \), which I call frequency, is actually non-angular wavenumber, so inverse wavelength.

The diagram in Figure 4b shows the experimental setup. Red and white lines depict the light path, while the dashed green line from the computer to the FTS indicates control and the other green line, from bolometer to computer, signifies the data pathway.
The reason for the existence of this expression is the fact we are dealing with a finite piece of material instead of the textbook situation of an infinitely deep dielectric. A finite chunk has two faces, which provide two surfaces off which light bouncing the slab can either reflect or refract. The equation for \( Y \) governs how much of it gets transmitted in the end after interfering between the faces an infinite number of times, getting progressively weaker.

We can generalize that equation to the case of a lossy dielectric. But first let us define the angular wavenumber \( k = 2\pi \nu \) in order to write a tighter expression, and also the optical impedance \( \eta \) as the reciprocal of the complex index. Then, the optical admittance is

\[
Y(k) = \frac{1}{\eta \cosh(k \, d) + \sinh(k \, d)}
\]  

(2b)

The cosine and sine morph into their hyperbolic counterparts, and all variables with complex versions change into those versions.

Now, we can construct the electric field amplitude reflection coefficient \( Y \) acts like an effective \( n \)—just compare to \( \rho \) for infinite slab:

\[
\rho = \frac{1 - Y}{1 + Y}
\]

(3a)

Then the corresponding power reflection is

\[
R \equiv \rho^2 = \rho \, \rho^*
\]

(3b)

Usually, the fractional intensities of reflected and transmitted light add up to 1:

\[
T + R = 1
\]

(4)

But if the material you are dealing with is capable of attenuating your signal, then the situation is slightly more complicated.

If \( I \) is the intensity of the radiation penetrating to a depth \( d \), then the part of the radiation undergoing interactions within a distance \( dl \) is given by

\[
I + \alpha + dl
\]

(5a)

where \( \alpha \) is defined as the attenuation coefficient. It is in units of inverse length because the further that radiation travels through a lossy medium, the greater the chance that it has of interacting with the atoms of that medium and thus end up not being transmitted by it.

The loss of intensity per \( dl \) is

\[
dl = -\alpha dl
\]

(5b)

If \( dl \) ranges from 0 to some length \( d \), and the initial intensity was \( I_0 \), then collecting like terms and antidifferentiating results in

\[
I = I_0 e^{-\alpha d}
\]

(5c)

Therefore, the translation between the non-lossy and lossy cases is simply

\[
T = 1 - R \Rightarrow T = (1 - R)e^{-\alpha d}
\]

(6)

The \((1 - R)\) part of the formula is essentially a sinusoidal function of frequency, taking into account the convergent infinite sum stemming from the Fabry-Perot interference: radiation traveling through a flat sample bouncing off its surfaces generating constructive and destructive interference in an alternating pattern, thus producing a sine wave. However, the exponential factor results in a decay of the oscillation. \( \alpha \) is an umbrella term taking into account the effects of both absorption and scattering phenomena.

The \( \alpha \) term is itself capable of being a non-trivial expression. In the past, experimenters like Halpern and others modeled it with a power law \((a + \nu^b)\), but such a functional form was an empirical convenience, and not a deep theoretical consequence [7,8]. But since that is not very intellectually satisfying, I instead attempted to fit my data to a physical model. I chose the Lorentz model, which treats electrons as masses attached to their respective nuclei via damped springs, since it gives one a theoretical basis for understanding attenuation [9]. Such a picture is realistic since electrons in a non-conductor are bound, not free as in metal. Lorentz accounts for absorption with energy-absorbing springs. High-frequency light is converted into oscillatory motion. Also, instead of unphysical parameters "a" and "b," the fit relies on three numbers that have meaning in the physical world. They are the resonance frequency at which absorption peaks, the spring damping coefficient, and the number of electrons participating in the resonance, that is, the actual number of particles doing the absorbing of the energy. Figure 5 shows that, in the frequency range in which I am interested, treating electrons as on springs approximates the empirical power law rather well.
Heald and Marion [9] derive formulae for the electron-spring model. I will now provide a summary of their results together with my own demonstration of how their $\alpha$ is reconciled with Halpern's $\alpha$ [7], which I must prove due to the fact they use their own sign conventions as well as slightly different conductivity definitions.

Heald and Marion write the complex index of refraction in terms of its real and imaginary components as

$$\hat{\epsilon} = \hat{n} = n(1 + i\kappa) \quad (7)$$

And, according to them, the spring model yields the following equations:

$$\text{Re}(\hat{\epsilon}) = n = 1 + \frac{2\pi}{\varepsilon_0} \frac{Nf_0e^2/m}{(\omega_0^2 - \omega^2) + 4\beta^2\omega^2} \quad (8a)$$

$$\text{Im}(\hat{\epsilon}) = -n\kappa = 4\pi N f_0\omega \beta e^2/m \frac{(\omega_0^2 - \omega^2)}{(\omega_0^2 - \omega^2)^2 + 4\beta^2\omega^2} \quad (8b)$$

This is a simplification from a more general case where these formulae are sums over several resonance peaks. I assumed only one peak, even though there may be many, since glass has a peak in the THz range that is larger than its neighbors [10]. $N$ is the number of electrons per unit volume participating in the absorption resonance, $f_0$ is the fraction of the total number participating in a particular resonance, but since I assumed one resonance peak, I set this equal to 1. $\omega$ is the radial frequency (in rad/s), while $\omega_0$ is the frequency at which resonance occurs. $\beta$ is the frictional damping factor, and $e$ and $m$ are the charge and mass of the electron or whatever the charge carrier is that resonates at $\omega_0$. Note that all this is a first-order Taylor approximation of the square root of the complex relative dielectric constant, epsilon. For the samples I tested, their dielectric constants may not have been small enough to make this a very good approximation, but computational difficulties forced me to keep only the first-order terms. Also, I assumed that the real part of the index of refraction did not vary significantly over the frequency range under experimental scrutiny. This remains valid as long as the resonance frequency is sufficiently high. I therefore left $n$ as a free parameter in fitting the data to the Lorentz model, in addition to $\omega_0$, $\beta$, and a factor I named $\kappa$ equal to $4\pi Ne^2/m$ that I created for the sake of greater simplicity. At least for my glass-filled mixtures, for which the glass was the main culprit responsible for attenuation, I could have tried to claim that $n$ equals the percentage of glass times the full-fledged expression for $n$ here plus a constant times the percentage of Teflon or other substrate, but that would not be correct if the wavelength of light is on the order of the glass inhomogeneities.

In this spring picture of the physics, we have the attenuation term $\alpha = \kappa n\kappa$, which is just the product of the imaginary part of the index of refraction and frequency. But this does not necessarily have to be exactly equal to the $\alpha$ in the exp(-$\alpha d$) found in Halpern [7], but it roughly is. Let us begin with how Marion defines the complex version of the dielectric constant:

$$\hat{\varepsilon} = \varepsilon + i \frac{4\pi \sigma}{\omega} \quad (9a)$$

where $\sigma$ is the DC electrical conductivity and $\omega$ is again the angular frequency.
The square root of this, which is equal to the complex refractive index, is, to first-order
\[ \sqrt{\varepsilon} = \sqrt{\varepsilon} \left( 1 + \frac{i \varepsilon \omega}{2 \varepsilon_0} \right) \] (9b)

This approximation is perfectly valid because glass, as well as the Teflon and other polymers in my experiment, were dielectrics, not good conductors \((\sigma / \omega \varepsilon_0 \ll 1)\). On the other hand, here is the same approximation using Halpern:
\[ \sqrt{\varepsilon} = \sqrt{\varepsilon} \left( 1 + \frac{\alpha}{2 \nu \varepsilon_0} \right) \] (9c)

Clearly, Halpern's \(\alpha\) equals the quotient of Marion's \(\alpha\) and negative 4\(\pi\). As a direct consequence, I can concisely prove that, indeed, \(\alpha = \nu \varepsilon_0\) works for Halpern, who defines \(\alpha\) as twice the real part of the complex angular wave-number, just the generalization of \(k = 2\pi / \lambda\).

I first compensate for the disparate sigmas by altering the imaginary part of \(n\)
\[ n(1 - i \varepsilon / (4\pi)) = n - nr \varepsilon / (4\pi) \] (10)

I then proceed with Halpern's definition of angular wave-number
\[ \kappa = i(k1 / \varepsilon_0) = i(k1 / n) = i(k1 / \varepsilon_0) / (n-1) \]
\[ = \kappa1 - \kappa \varepsilon / (4\pi) = 2\nu \varepsilon_0 \varepsilon_0 + \pi \varepsilon_0 / 2 \quad \alpha = \nu \varepsilon_0 \]
and the attenuation terms fall right out. In conclusion, I assert that
\[ T(v) = (1 - R(v))e^{\text{loss}} \] (11)
characterizes fully the transmission coefficient as a function of frequency, given a frictional factor, a resonance frequency, and a particle information term \((4\pi \varepsilon_0 n^2 / m)\) to plug into the dimensionless \(\kappa\), the imaginary part of the index, which handles the lossiness.

The situation is a bit more complicated in another sense, though. I must compensate for the fact my apparatus had a finite angle of acceptance.

All the math above is applicable only if one considers a bundle of light rays normal to each surface. If rays enter in from non-zero angles to the normal, then \(T(v)\) has to be found separately for every angle. The final transmission is the average, as described in Timusk and Richards [11]. I am forced to replace the “\(d\)” inside \(e^{-\text{loss}}\) with \(d \sec(\theta)\) because at an angle \(\theta\) light must propagate along this greater distance. The “\(d\)” in the Fabry-Perot admittance, though, is replaced with \(\cos(\theta)\). The interfering waves thus see a thinner substance.

I had to find the most restrictive Winston cone in my setup in order to know what the limiting maximum angle was. For most of my room temperature data, this was the Winston cone looking down into the FTS, but for the cold data, it was the cone connected to the blackbody because for my cold data I removed the cone looking into the FTS to increase my signal strength. I put the bolometer directly on the FTS for the majority of the 300 K measurements, with a sample right under, in the light path, the cryogenic filter holder being unnecessary and causing some attenuation with all the turns rays have to take through it. In order to make more sensible comparisons, however, any samples I tested at 4.2 and/or 77 K in the glass dewar, I also tested in the dewar even at 300 K.

If \(r_\ell\) is the radius of the most restrictive Winston cone at its center and \(r_\ell\) is the radius at its end, then the following relation holds:
\[ A\Omega = \text{constant (product of area and solid angle)} \]
\[ \Rightarrow \pi r_\ell^2 = \pi r_\ell^2 \frac{d}{\max} \sin(\theta) \cos(\theta) d\theta \] (12a)
\[ \Rightarrow \pi r_\ell^2 \left( \frac{d}{\max} \right) \frac{\sin^2(\theta) \cos(\theta)}{2} \]
\(\pi\) is the solid angle subtended by the circular hole at the center of back-to-back cones, so I have set up a relationship here to extract the maximum half-angle for the total solid angle encompassed at the mouth of the cone. The transmission must be properly averaged over \(A\Omega\) ranging from zero half-angle to the max half-angle. The final answer is thus
\[ T(v) = \frac{\int_0^{d/\max} T(v, \theta) \sin(\theta) \cos(\theta) d\theta}{\int_0^{d/\max} \sin(\theta) \cos(\theta) d\theta} \] (12b)

Although there is one more note: theta-max must be the angle in the test sample, not the angle of incidence in air. So, if \(n\) is the index of that sample, Snell's Law states that this would be \(\sin^{-1}(\sin(\theta_{\max}) / n)\). Also, since the full functional form of \(T\) (eq. 11 with eq. 8b and many others imbedded within it) is not reasonable to integrate analytically, I computed the numerator of 12b numerically. Note the sine, whose growth with angle keeps track of the ability of radiation to bombard a surface in more ways at steeper angles. But, it is forced to spread over larger area, hence the cosine.
IV. DATA

Before reviewing my results, I will first give an outline of how I processed my raw data in order to produce a transmission spectrum.

The raw interferogram is a graph of intensity (in units the bolometer uses—their meaning in an absolute sense is inconsequential in the sense that I only look at relative numbers in the end) versus distance, in terms of a grid inside the FTS ranging from -120,000 to 120,000 with data being taken only every 4 points. The large structure in the interferogram is the white-light fringe, which is the result of constructive interference of all frequencies at a particular distance. By distance in an interferogram picture, I mean a measure of the optical path difference. I subtract the mean of an interferogram from it, so that oscillations of constructive and destructive interference are centered at zero intensity. I then apply a Hanning window filter function to the resulting interferogram, which causes the graph to taper off at the edges. I do this in preparation for taking a Fourier Transform, so that the computer does not encounter a difficulty with taking the Fourier Transform of a finitely-long function with a sudden drop to zero at its edges.

I take the full-fledged complex, but discrete Fast Fourier Transform (FFT) of the interferogram, and then take its absolute value. The interferogram should, strictly speaking, be a symmetric function (about the white-light fringe) and thus have a real Fourier Transform, the imaginary part being only noise in the data. But this is not the case because the bolometer houses its own filter. This means that the bolometer does not record "true" data in the sense that it filters it through some transfer function. In my data, I counted on being able to disregard the impact of the bolometer's transfer function due to the fact that in order to calculate transmission spectra I took the quotient of two FFTs which should both have been affected in the same way. I plot the FFT of the raw data versus the frequency in inverse cm. In order to translate the units of displacement in the FTS into frequency I needed the following facts. FTS Grid points are 50,000 per cm in distance that the carriage with the dihedral mirrors moves. Now converting to the distance that the light moves gives an extra geometrical factor of 4*cos(30). Then I had to take into account the fact that each datum is 4 grid points apart. The result was that

\[ v_i = i \times 50,000 \text{ grid points per cm} \\
(4 \text{ grid pts/data pt})(4\cos(30)) N \]

where \( N \) is the total number of data points and \( i \) is an integral index running from 0 to half the number of data points.

![Figure 6. The initial steps in the data-taking process.](image)
I divide the FFT of the interferogram of a sample by a reference interferogram in order to obtain a transmission spectrum, a picture of the transmission coefficient between 0 and 1 versus frequency. However, every data-taking run of the FTS yields two sets of data, one for the dihedral carriage moving left, another for when it is moving right. In addition, I operated the FTS with the carriage moving at different speeds, 2.0 optical cm/s and 0.13 cm/s, the reason being the fast speed provided good data at low frequencies and the slow speed provided good data at high frequencies (I will give the reason in my noise explanation.) Thus, for each sample, I generated four sets of data: fast left, fast right, slow left, and slow right. But I also took a reference both before and after every one sample because the bolometer housed in the gold dewar featured in Figure 4a suffered from gain variation. In sum, then, my full process of data analysis was:

Compute... (for one particular speed)

1. average (arithmetic mean) of left intensities (FFTs) for the 2 references;
2. FFT of averaged left reference and averaged right reference
3. average of FFTs in (2.)
4. average of left and right sample spectra
5. result of (3.)/(4.)
6. repeat steps (1.)-(5.) for other speed

Next comes the combination of the data from the slow and fast optical speeds.