6.1 *uniform acceleration* It is possible to expose a particle to a constant force, as seen from its own rest frame. In that frame, the particle experiences uniform acceleration: its speed changes by a constant amount for each (small) unit of proper time. In the particle's frame, the acceleration is always a, while its own speed is always 0. Now suppose the particle starts from rest in the lab frame and starts "falling" under uniform acceleration. It accelerates for a proper time τ, during which a clock in the particle frame would tick n times.

a) How does the rapidity of the lab frame change with the proper time of the particle?
b) The ticks of the particle's clock constitute events in any frame of reference. How many of these ticks occur in the lab frame. (I'm not speaking of a lab clock here.)
c) How much lab time has elapsed after the k'th tick?
d) How does the particle's speed increase with time according to a lab observer?
e) Nonrelativistically, the speed will reach c in a proper time $\tau = c/a$. How fast is it really going after this much time?

6.2 *relativistic collision cf. Griffiths 10.30* A particle of mass m whose total energy is twice its rest energy collides with an identical particle at rest. If they stick together, what is the mass of the resulting composite particle? What is its velocity? Use the conservation of 4-momentum and its Lorentz length.

6.3 *colliding beams, cf. Griffiths 10.32* Present-day accelerators can accelerate protons of mass m to an energy $E = \gamma mc^2$, where $\gamma \simeq 1000$.

a) What is the center-of-mass energy when such a particle collides with a like particle at rest? Do it for general γ; then evaluate for $\gamma = 1000$.
b) Now suppose two oppositely-moving particles with this same energy collide. What is the center of mass energy? What is the ratio of this center of mass energy to that found in part a), as a function of γ?