7.1 *relativistic Lorentz force* As Griffiths says in Eq. 10.138, a relativistically consistent form of the Lorentz force law is

$$dp^a/d\tau = q\eta_b F^{ab}$$

where p^a is the a component of the four-momentum. Repeated indices (*viz.* b) are to be summed. Here $\underline{\eta}$ is the four-velocity and η_b just means $\eta^a g_{ab}$ and $g_{ab} = g^{ab}$ is the ab component of the metric matrix defined earlier. The quantities F^{ab} make up the electromagnetic field tensor: $F^{0i} = -F^{i0} = E_i$ and $F^{ij} = -F^{ji} = \epsilon_{ijk}B_k$. The η_a are the components of the four-velocity.

a) According to this formula, what is the derivitive of the $a = 1$ component of $\underline{p}$ with ordinary time t in some arbitrary frame of reference, in terms of the conventional speed $\vec{\mathbf{v}}$ in that frame and the $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ fields in that frame? Compare with the Lorentz Force formula from the back cover of Griffiths.

b) According to this formula, what is the derivitive of the $a = 0$ component of $\underline{p}$ with ordinary time in some frame where the velocity is $\vec{\mathbf{v}}$, the electric field is $\vec{\mathbf{E}}$ and the magnetic field is $\vec{\mathbf{B}}$? How does the mechanical energy vary with time according to the classical Lorentz force law?

7.2 *how fields transform the rest frame* Over a short interval of proper time $\Delta\tau$ the electromagnetic field changes the state of motion of a charged particle. Using the relativistic Lorentz law from the previous problem, express the transformation of $\underline{\eta}$ in the form

$$\underline{\eta}\big|_{\tau+\Delta\tau} = \mathsf{S}\,\underline{\eta}\big|_{\tau}\,.$$

(To construct this transformation you must divide the left side by m to get a four velocity; you must also convert the superscript on the left to a subscript, to match the η_b on the right.)

a) Find the form of the matrix S. Evidently, S tells how the rest frame of the particle transforms with time. Does this transformation preserve the Lorentz length?

7.3 *Thomas precession* In some lab frame, there is a uniform B field in the 3 direction. According to the previous problem, the rest frame of a particle with mass m and charge q rotates with time.

a) Find the angular velocity of this rotation, and the rotation axis.

b) If the particle has initial momentum $\underline{p}$, how much proper time elapses before $\underline{p}$ returns to its intial direction?

c) How much time has elapsed in the lab frame? Compare with Griffiths Eq. 5.4.