8.1 *Symmetry preservation REVISED* The tensor transformation law preserves the symmetry of a tensor under interchange of two indices, as this problem will show. The tensor transformation law is given by Griffiths Eq. 10.47 for one-index tensors. Any product of four vectors such as $A^a B^b C^c D^d E^e$ must transform $A...E$ into the primed frame, by contracting each index with a Λ Lorentz-transformation tensor. The same is true for any sum of such products. Denoting the quantity as t^{abcde}, this means that t transforms according to the rule $(t')^{\alpha\beta\gamma\delta\epsilon} = \Lambda^\alpha{}_a \Lambda^\beta{}_b \Lambda^\gamma{}_c \Lambda^\delta{}_d \Lambda^\epsilon{}_e t^{abcde}$. As with vectors, a lower index just denotes contraction with g_{ab}, so that $t^{ab}{}_c{}^{de}$ just means $g_{c\gamma} t^{ab\gamma de}$. Suppose an arbitrary tensor t^{abcde} is symmetric under exchange of the second and 4th index, *viz.*b and d.

a) Use the tensor transformation law to show that the transformed tensor t' must also be symmetric in the 2nd and 4th index. The same reasoning shows that symmetry *or anti*symmetry in any pair of indices is preserved, provided both indices are up (or both are down).

8.2 *The antisymmetric tensor* The antisymmetric tensor ϵ_{abcd} is defined by i) $\epsilon_{0123} = 1$ and ii) ϵ is antisymmetric in all adjacent pairs of indices. (Jackson uses the opposite convention for his ϵ.)

a) Show that $\epsilon = 0$ if any two indices are equal.
b) What is the value of ϵ_{3201}? ϵ_{1320}?
c) What is the value of ϵ^{0123}?
d) OPTIONAL. Show that all the elements of ϵ_{abcd} remain invariant under an infinitessimal (or a finite) Lorentz transformation. That is, $\epsilon'_{abcd} = \epsilon_{abcd}$.
e) The dual tensor G_{ab} defined in Griffiths may be written $G_{ab} = \frac{1}{2}\epsilon_{abcd} F^{cd}$ except for signs. Verify this for one E component and one B component and thus determine the sign differences between Griffiths' G_{ab} and $\frac{1}{2}\epsilon_{abcd} F^{cd}$.

8.3 *Invariants of the field tensors* If two Lorentz tensors are multiplied and each upper index is contracted (summed) with a lower index, the result is a Lorentz scalar. This means one can construct Lorentz scalars out of the electromagnetic field tensor F^{ab} and the dual tensor G^{ab} defined in Griffiths 10.129 and 10.130.

a) Using the definitions of F^{ab} and F_{ab} find the Lorentz scalar $F^{ab}F_{ab}$ in terms of E and B.
b) Do the same for $G^{ab}G_{ab}$.
c) Do the same for $F^{ab}G_{ab}$.
d) If E is perpendicular to B in one co-ordinate system, must they be perpendicular in all?
e) If $|E/c| < |B|$ in one co-ordinate system, must the corresponding statement be true in all?

8.4 *Maxwell stress* A particle of mass m and charge q is suspended below an equal and opposite charge so that the Coulomb attraction just balances its weight. The force to hold the particle up is transmitted by the fields. This force must pass through the horizontal midplane, half way between the charges. Suppose this midplane is a distance d from each charge.

a) What fraction of the weight passes through a circle of radius d on the midplane?
b) What size circle on the midplane transmits half the weight?

8.5 *Complex intensity (cf Griffiths 8.11)* The text adopts the convenience of representing sinusoidal waves as the real part of a complex wave, *e.g.*$E = \mathrm{Re}(\tilde{E})$. In calculating intensities, one needs to know products of fields, such as EB averaged over a cycle. Such products can be expressed in terms of the complex fields, as well. Suppose $\tilde{F}(\mathbf{r}, t)$ is a complex-valued wave of the form $\tilde{f}(\mathbf{r})e^{i\omega t}$ and $\tilde{G}$ is another one of the form $g(\mathbf{r})e^{i\omega t}$.

a) Find an expression for the average of FG over a cycle in terms of $\tilde{F}\tilde{G}^*$, where * indicates complex conjugation. (Note that if F and G had different frequencies, their average product would vanish. Such waves are *incoherent*.)
b) Use this trick to express the average energy density $\langle u \rangle$ and the average energy flux $\langle \mathbf{S} \rangle$ in terms of the complex fields $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{B}}$.