s.1 *dark mirrors* Some metals, like lead, are darker than others. This suggests that they reflect a smaller fraction of the light that hits them.

a) What should be the skin depth of a metal that reflects half the light that hits it, assuming $\epsilon = \epsilon_0$ and $\mu = \mu_0$? You may assume the wavelength in vacuum is 500 nm.

s.2 *evanescent waves* You know from previous studies that when a wave passes into a medium with a higher wave speed ($c_2 > c_1$), Snell's law may have no solution. At angles θ_I such that $\sin(\theta_I) > c_1/c_2$, there is no angle θ_T that would satisfy Snell's law. This means there is no transmitted wave. The wave is completely reflected. But this does not mean there are no fields in medium 2. It only means that it is not a sinusoidal wave of the usual form.

a) Consider a wave in which $E = E_0 \exp(i\kappa \cdot r - i\omega t)$. Suppose κ_x is complex, while the other two components of κ are real. In a medium with wave speed c, what is the condition on κ_x such that the wave equation is satisfied for a given, real, $\kappa_\perp$ and ω.

b) The interface between medium 1 and medium 2 is in the x direction. Find κ_x in medium 2 when there is a wave incident from medium 1 with speed c_1 and angle θ_I. Don't assume $\sin(\theta_I) < c_1/c_2$.

c) Some of the solutions you found in b) have E decaying exponentially as one goes into Medium 2. What angle θ_I gives the fastest decay, *i.e.,*the shortest decay length? What angle θ_I gives a decay length twice as long as the shortest one? What happens as θ_I approaches the critical angle, at which $\sin(\theta_I) = c_1/c_2$?

d) If the incident E is in the plane of incidence, what is the direction of E in medium 2 when the wave is decaying exponentially there?

s.3 *magnetic dipole radiation* A magnetic dipole along the z axis is oscillating at frequency ω, so that the dipole moment $\mathbf{m}(t)$ has the form $\hat{k} m_0 \cos(\omega t)$ (Cf. Griffiths 9.1.3). Suppose this dipole is a current loop of radius a, and that $a \ll \lambda = c/(2\pi\omega)$.

a) Find the vector potential $\mathbf{A}(r, \theta, t)$ for points $r \gg a$, so that only the dipole contribution of the the loop is important. But don't assume that $r \gg \lambda$. This amounts to re-deriving Griffiths Formula 9.46

b) Find the divergence of $\mathbf{A}$. Is the Lorentz gauge condition relating V and $\mathbf{A}$ satisfied?

c) Find the E and B fields.

d) Find the time-average energy current $\langle S \rangle$. Compare with Griffiths 9.52, which assumed that $r \gg \lambda$.

s.4 *radiation and acceleration* In an oscillating electric dipole, the E and B fields depend only on the charge and current at the retarded time. Suppose the dipole moment is caused by a charge moving back and forth across a stationary opposite charge.

a) Express the E field at a distant point r in the radiation zone in terms of the particle's velocity $\dot{z}$ and acceleration $\ddot{z}$ instead of the frequency and the dipole moment. Since this is an oscillating dipole, you may use Eq. 9.31. Compare with Eq. 9.107, the explicit formula for a moving charge. Do these formulas agree as they should?