Physics 316, Problem Set 4, due Tuesday October 25 in class

1. Earth-Venus highway In the year 2145, in response to an economic recession, the government launches a project to build a bridge between Earth and Venus. The bridge will be made of modern materials so that its mass will be negligible compared to that of either planet. It will make a rigid connection between Earth and Venus so that their distance will be fixed at their current distance $2L$ of closest approach. However, doom-sayer scientists decry the project, saying that it will perturb the orbit of the earth. Assess this claim, taking the Earth's radius to be R, and both masses to be m. Consider both orbits to be circular before the planets are joined. Treat both planets as point masses and neglect any effects due to rotation of the planets about their axis. (The bridge is to be installed so that it doesn't interfere with this rotation). Take as co-ordinates the radial distance r from the sun to the center of the bridge, and its orbital angle ϕ, and the angle θ that the bridge makes with the radial direction. It is convenient to express times in units of the earth year τ.

a) Express the kinetic energy T in terms of the given co-ordinates, with the help of the rigid-body expression $T = T_{cm} + \sum_i m_i r_i \omega^2$. Here T_{cm} is the kinetic energy of the center-of-mass motion m_i is mass of the ith particle in the body, r_i is its distance from the center of mass and ω is the angular velocity of the particle relative to the center of mass.
b) What is the Lagrangian of the system as a function of r, ϕ, and θ?
c) Find a cyclic co-ordinate and an associated conserved function of r, ϕ and θ.
d) State the equations of motion for r, ϕ, and θ.
e) Find the Hamiltonian and Hamilton's equations of motion.
f) From the statement of the problem, give initial conditions sufficient to solve the subsequent motion.
g) Discuss whether θ will oscillate back and forth within a given range, or will it range over 2π. I. e. will the earth remain beyond Venus or rotate around Venus?