**Problem 1.1 (5 points) Orthogonalizing vectors (cf Taylor [1.6])**

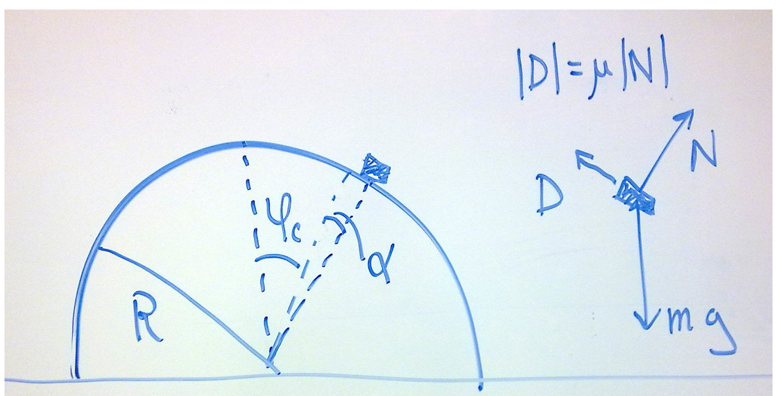
Vector $\vec{A}$ is defined as $\hat{x} + 2\hat{y}$. Vector $\vec{B}$ is defined as $\hat{x} + s\hat{y}$, where $s$ is some constant. For what value(s) of $s$ would $\vec{B}$ be orthogonal (*i.e.,* perpendicular) to $\vec{A}$?

**Problem 1.2 (10 points) Time-determined force**

A mass $m$ lies at rest at position $x = 0$ on a frictionless, horizontal track. A string is attached to the mass. The other end of the string is attached to a force actuator that can exert a specified horizontal force $F$ as a function of time $t$. We program the actuator to exert a force $At$. (Naturally any real actuator could only exert such a force over a limited range of $t$.)

a) What is the equation of motion governing the position $x(t)$?
b) By integrating this equation once, find the velocity $\dot{x}(t)$.
c) By integrating the equation for $\dot{x}(t)$ find the position $x(t)$.
d) If the mass moves one inch in a given time $t_0$, how far will it go in time $2t_0$?
e) If the mass is doubled and the experiment is repeated, how far will it go in time $t_0$?

**Problem 1.3 (10 points) Frictiony sphere**




By combining partial information about the forces in different directions, one can find the motion along a given path.

A small mass $m$ placed on a frictionless sphere at some angle $\phi$ from the vertical will accelerate downwards because of gravity. However with friction there is a region near the top where the mass does not accelerate. If given a small velocity, it slows down and stops. It only speeds up if the angle from the vertical is $\phi_c$ or more.

a) Find the equation of motion for $\ddot{\phi}$ analogous to Eq. 1.50. It can be given explicitly in terms of $\phi$, $\dot{\phi}$, the (kinetic) coefficient of friction $\mu$, the radius $R$ and the gravitational acceleration $g$.
b) Determine the dynamic friction coefficient $\mu$ in terms of $\phi_c$.

**Solution:**

**1.1**

$\vec{A} \cdot \vec{B} = 1 + 2s$. If $\vec{A}$ and $\vec{B}$ are to be orthogonal, we must have $0 = \vec{A} \cdot \vec{B} = 1 + 2s$ therefore $s = -1/2$.

**1.2**

a) $m\ddot{x} = At$

b)

$$\dot{x}(t) = \dot{x}(0) + \int_0^t dt' \; \ddot{x}(t') = 0 + \frac{A}{2m}t^2$$

c)

$$x(t) = x(0) + \int_0^t dt' \; \dot{x}(t') = 0 + \frac{A}{2m}\frac{1}{3}t^3$$

d)

$$x(2t_0)/x(t_0) = (2t_0)^3/{t_0}^3 = 8; \quad \text{thus, } x(2t_0) = 8x(t_0) = 8 \text{ in}$$

e)

$$x(2m, t_0)/x(m, t_0) = \frac{1}{2}; \text{thus, } x(2m, t_0) = \frac{1}{2} \text{ in}$$

**1.3**

a) The equation of motion is

$$mR\ddot{\phi} = mg\sin\phi - |D| \tag{1}$$

where $|D| = \mu N$. Since there is no velocity in the radial direction, $\vec{v} = v\hat{\phi}$ and

$$\dot{\vec{v}} = \frac{d}{dt}(v\hat{\phi}) = \dot{v}\hat{\phi} + v\dot{\hat{\phi}}$$

As explained in the text, $\dot{\hat{\phi}} = -(v/R)\hat{r}$. thus the equation of motion in the $\hat{r}$ direction reads

$$-m(v^2/R)\;\hat{r} = -mg\cos\phi\;\hat{r} + N\;\hat{r}$$

so that $N = mg\cos\phi - mv^2/R$ or $N = mg\cos\phi - m\dot{\phi}^2R$
Combining, the equation of motion (1) becomes

$$mR\ddot{\phi} = mg\sin\phi - \mu\;(mg\cos\phi - m\dot{\phi}^2R)$$

b) The critical angle $\phi_c$ is that at which $\ddot{\phi}$ goes through zero with arbitrarily little motion *i.e.*, $\dot{\phi} \to 0$:

$$0 = mg(\sin\phi_c - \mu\cos\phi_c)$$

Thus $\mu = \sin\phi_c/\cos\phi_c = \tan\phi_c$.