Physics 361 **Problem set 3** **due** *Wednesday* **October 30, 2002**

P4.1 Poor-man's localization In the alternating-spring system of Figure 22.9, there is a gap in the density of states—*i.e.,* a frequency range where there are no normal modes. Still, we can excite the system at any frequency we like, including a frequency in the gap. Consider a semi-infinite chain like Fig. 22.9 with $0 < K - G \ll K$, so that the gap is small. Now suppose that the end mass is oscillated with a frequency ω in the middle of the gap, at frequency $\sqrt{(K+G)/M}$.

a) Find a solution to Eq. 22.37 for this frequency, but complex wavevector k.
b) Use this solution to determine the behavior of u_1 and u_2 of Eq. 22.34 as a function of distance from the end.
c) How does the penetration depth grow as $G \to K$ and the gap disappears?

P4.2 Inelastic scattering A bravais lattice of sites $\vec{R}$ is vibrating with a normal mode that has a small displacement amplitude u_0, wavevector $\vec{k}$ and frequency ω. Thus $\vec{u}(\vec{R}) = \vec{u}_0 \cos(\vec{k} \cdot \vec{R} - \omega t)$. A beam of radiation is scattered from this lattice with a change of wavevector $\vec{q}$. As told in the handout `scattering.pdf`, its amplitude at the detector is given by

$$\psi(t) = \text{const. } e^{-i\Omega t} \sum_{\vec{R}} \exp\left(i\vec{q} \cdot \vec{r}(\vec{R}, t)\right),$$

where Ω is the frequency of the incident wave and $\vec{r}(\vec{R}, t)$ is the position of the $\vec{R}$ atom at time t.

a) Find the wavevectors q that are coherently scattered as a result of the vibrations. The assumed smallness of u_0 means that $qu_0 \ll 1$ for all q of interest.
a) What are the changes in frequency of the scattered waves in a)?
c) Suppose that the vibrational wave is propagating straight towards the beam, and that the beam particles have a much higher frequency than the vibrations: $\Omega \gg \omega$. The crystal lattice is not alligned with the beam. Find the frequency of the wave scattered back towards the beam.