P5.1 surface plasmon and thin-film plasmon Charge-oscillation modes occur at the surface of an electron gas. The best known one is treated in A&M, page 27, problem 5. In this problem one finds the dispersion relation for charge oscillations of wavevector q along the surface of an electron gas with density n in which the fields die off exponentially with distance on either side of the interface: $q^2 = (\omega/c)^2\ (\omega_p^2 - \omega^2)/(\omega_p^2 - 2\omega^2)$.

a) Sketch ω *vs.* q and find the limiting $\omega(q)$ for $q \ll \omega_p/c$ and for $q \gg \omega_p/c$.

Consider a thin layer of electron gas with n_s electrons per unit area. As usual, there is a compensating uniform background charge. Suppose that the electrons are displaced from their initial uniform state by an amount $\mathbf{u}(x) = u_0\hat{\mathbf{x}}\sin qx$. This produces a nonuniform charge density profile $n_s(x)$

b) Find the charge density profile $n_s(x)$.

c) Find the electrostatic potential $\phi(x)$ resulting from this charge density. For this it is helpful to know the electrostatic potential $\phi_l(x)$ from a line of unit charge density and then express $\phi(x) = \int dx\phi_l(x' - x)n_s(x')$.

d) Find the electrostatic potential energy per unit area resulting from $u(x)$.

e) Find the kinetic energy per unit area if u_0 is time dependent.

f) Using parts d and e, find the oscillation frequency for the charge as a function of q.

g) For a thin film of metal with thickness $h \ll 1/q$, how does the frequency in f) compare to the conventional surface plasmon frequency in a) for $h\omega_p/c \ll 1$ and $h\omega_p/c \gg 1$.

h) Do you have any comments to make about the group velocity of the thin-film mode as $q \to 0$?

P5.2 Phonon and electron specific heats Chapter 2 notes that electronic specific heat dominates over phonon specific heat at low temperatures. However , the phonon specific heat grows faster with temperature, so that there is a temperature $T_\times$ where the two are equal. The electron thermal energy is always much below k_BT per atom, so at $T_\times$ the phonon specific heat must be far below the classical Dulong-Petit value. Relative to the fermi energy $\mathcal{E}_f$, this $k_BT_\times$ can be expressed in terms of the fermi velocity v_F and the speed of sound c of page 457.

a) Find this expression for $T_\times$.

b) How big is $T_\times$ for sodium? For copper? For this you have to use γ values from Chapter 2 and Θ_D values from p. 461.

P5.3 Friedel oscillations Chapter 2 warns that on length scales of order $1/k_F$, the quantum properties of the electron gas become significant. One such property is the total density near a wall. Consider an ideal Fermi gas near a hard, flat wall. At the wall, the density must vanish; thus, each electron wave-function $\psi_k(x)$ must vanish. The total density $n(x)$ at distance x from the wall is the sum of densities of the electrons.

a) Find the density $n(x)$ for a one-dimensional ground-state Fermi gas with fermi-momentum k_F. Here $\psi_k(x) = \text{const.}\ \sin kx$.

b) For a three-dimensional ground-state Fermi gas with Fermi momentum k_F, find the density $n(x)$ at distance x from a planar wall. Here $\psi_{\mathbf{k}}(\mathbf{r}) = \text{const.}\ \sin(k_x x)e^{ik_y y + ik_z z}$.

c) Graph these two functions.