P6.1 Electrical conductivity of a Fermi gas The Drude formula for the electrical conductivity is based on Eq. 1.24 for the average momentum of the electrons. In this problem we examine the validity of this equation for a fermi gas. Suppose one has independent, free electrons in the ground state, with fermi energy $\mathcal{E}_F$. Thus the occupation number or probability that a momentum state $\mathbf{p}$ is occupied is $\Theta(\mathcal{E}_f - \mathbf{p}^2/(2m))$, where Θ is the unit step-function. Consider a single-particle wavefunction with initial momentum $\mathbf{p}$. If the electron is in a uniform field E in the x direction, the state after time t is also an eigenstate of momentum.

a) What is its momentum $\mathbf{p}(t)$ in terms of $\hbar$, its mass m and its charge e?
b) If all the electrons in the ground state are subjected to this field for a time t, what is the occupation number $f_t(\mathbf{p})$, and what is the average momentum $\langle \mathbf{p}(t) \rangle$?

Now suppose that the electric field is turned off and another perturbing Hamiltonian $\mathcal{H}'$ is turned on. This perturbation operates on any single-electron state and produces a random superposition of momenta with a distribution of energies $p(\mathcal{E}) = \text{(constant)}\ e^{-\beta\mathcal{E}}$ We take $1/\beta \ll \mathcal{E}_f$. This $\mathcal{H}'$ can only operate if the chosen final momentum state is unoccupied. Therefore the rate of transitions into state $\mathbf{p}_f$ from any initial state is (constant) $e^{-\beta\mathcal{E}(\mathbf{p}_f)}(1 - f_t(\mathbf{p}_f))$, where f_t is the occupation number distribution found in b) and $\mathcal{E}(\mathbf{p}) \equiv \mathbf{p}^2/(2m)$. This $\mathcal{H}'$ causes the average momentum to change with time.

c) If an electron with momentum $\mathbf{p}$ is scattered by this process, what is its average final momentum? You can assume that the electric field of part a) was weak, so that $f_t(\mathbf{p})$ is only slightly different from the equilibrium distribution $f_0(\mathbf{p})$
d) If an occupied electronic state is chosen at random and undergoes scattering as in c), what is the average change of momentum $\Delta \langle \mathbf{p} \rangle$?
e) How does the rate of change of $\langle \mathbf{p} \rangle$ owing to this scattering depend on $\langle \mathbf{p} \rangle$ itself? You may ignore prefactors since we have said nothing about the intrinsic strength of the scattering.
f) Considering the electric field effect of a) and b) and the scattering effect of e) comment on the applicability of Eq. 1.24 to the quantum case.

P6.2 Thermocouple The most obvious experiment to see the thermopower of a metal is to attach the ends of two wires made of different metals and then subject the two ends to the same temperature difference as shown in Fig 13.1. If one measures the open-circuit potential across in this current loop, one gets a voltage proportional to the temperature difference. This voltage is also proportional to the difference in thermopowers of the two metals. This effect provides a means of measuring temperature differences. Such a device is called a thermocouple.

a) Predict the voltage per degree of temperature difference for two ideal Sommerfeld metals with Fermi energies $\mathcal{E}_1$ and $\mathcal{E}_2$.
b) What pair of metals in Table 2.1 should give the biggest voltage, and how big is it for a one-degree temperature difference?
c) The thermopower as predicted in Eq. 1.59 depends on the specific heat. Considering the measured specific heats in Table 2.3 rather than the ideal ones, how much does your answer in b) change? Using this way of calculating, what is the best pair of metals in Table 2.3?
d) The Handbook of Chemistry and Physics reports thermocouple coefficients for copper and constantin wires and for iron and constantin wires. Deduce the difference in thermopower of iron and copper and compare with your predictions using Table 2.1 and Table 2.3.

P6.3 reflecting and transmitting bandstructure Cf. Problem 8.1. A one-dimensional crystal consists of a row of equally spaced atoms. These interact with electrons to form a periodic potential like that shown in Figure 8.4. Considering a single atom, an electron with energy $(\hbar K)^2/2m$ incident from the left will scatter from this potential. The transmitted wave has amplitude $t = |t|e^{i\delta}$, and the reflected wave has amplitude $r = \pm\sqrt{1 - |t|^2}e^{i\delta}$. The same r and t hold for waves incident from the right. r and t depend on the incident energy in general. (The book suggests how to prove that r and t are related this way.) If the spacing between these potentials is a, there is a definite relation between the Bloch wavevector k and the "free wavevector" K defined above.

a) Determine $\cos ka$ as an explicit function of K, $|t|$ and δ.