Long-lived tendrils in viscous entrainment

Wendy W. Zhang
Laura E. Schmidt*

Physics & James Franck Institute
University of Chicago

Geophysical Fluid Dynamics Session
AGU meeting San Francisco 2008

*Now at Physics of Fluids group (Lohse) U. Twente Netherlands
Experiment on thermal convection in heterogeneous liquid

- Lighter & more viscous layer overlying
denser & less viscous liquid

- Cool from top
- Heat at bottom

Jellinek & Manga Nature 2002
Experiment on thermal convection in heterogeneous liquid

Lighter & more viscous layer overlying denser & less viscous liquid

Cool from top
Heat at bottom

Thin tendril of liquid from lower layer

Jellinek & Manga Nature 2002
Tendrils persist until lower layer drains

- Tendril entrained by warm upwards flow in thermal boundary layer
- Entrainment stabilizes upwards flow

Kumagai et al. 2008
Geophysical Res. Lett.

Jellinek & Manga 2004
Review of Geophysics
Fluid dynamics question: What sets tendril thickness?

Steady tendril far thinner than layer depth or typical lengthscale in convecting flow
**Geophysics Question:** Tendrils as analog of hot spots?

Many convection regimes

Increasing stratification (Bond number)

Persistent tendril requires strongly stratified layers

Gonnermann, Jellinek & Manga 2004
Earth Planet. Sci. Lett

Kumagai et al. 2008
Geophysical Res. Lett.
Tendril vs spout

Steady axisymmetric withdrawal in viscous, immiscible layers
Above finite threshold, lower-layer liquid entrained as steady spout

Blanchette & Zhang 2008
**Very thin spout forms when transition to hump approaches**

Does thin tendril form near an analogous transition?

A possible hitch: hump state is never stable

Hydrostatic & viscous stress alone cannot stabilize interface against short-wavelength disturbances forming tendril

Unstable hump controls flow pattern in lower layer
Tendril draws liquid from thin layer above hump profile

\[ U_r = -\frac{E r}{2} \quad U_z = E_z \]
\[ E \sim \frac{F}{2 \pi \mu S^2} \]

Schmidt & Zhang 2008
\[ U_r = -\frac{E r}{2} \quad U_z = E_z \]

\[ E \sim \frac{F}{(2 \pi \mu S^2)} \]

\[ \mu E \sim \Delta \rho g H \]

Schmidt & Zhang 2008
\[ U_r = -\frac{E r}{2}, \quad U_z = E z, \]
\[ E \sim \frac{F}{2\pi \mu S^2}, \]
\[ \mu E \sim \Delta \rho g H, \]
\[ E H \sim \Delta \rho g \frac{R^2}{\mu_0}, \]
\[ R \sim \left(\frac{\mu_0}{\mu}\right)^{1/2} H, \]

Schmidt & Zhang 2008
\[ R \sim (\mu_0 / \mu)^{1/2} H \]

*Tendril thin if $H$ small (strong stratification)*

*Tendril thin if lower layer less viscous than upper layer*
Unique tendril selected by matching 2 parts of interface

long & slender
tendril
NONLINEAR

weakly deflected
interface
LINEAR

Schmidt & Zhang 2008
**Governing equation for tendril shape**

\[
Q_{\text{tendril}} = 2\pi \left[ \frac{E_z R^2(z)}{2} - \frac{R^4(z)}{16 \lambda \mu} \frac{d}{dz} \left[ 2\mu E \left( 1 + \frac{z}{R(z)} \frac{dR}{dz} \right) + \Delta \rho g z \right] \right]
\]

- **flow rate unknown**
- **exterior viscous effect**
- **pressure gradient due to flow inside tendril**
- **gravity**

**Boundary conditions**

At base, merge into hump

\[
R(z) \to R_s(z) = B \left( \frac{\ell_z}{z} \right)^{\alpha} e^{-z/\ell_z} \quad \text{as } z \to 0
\]

Far away, liquid steadily entrained

\[
R(z) \to R_\infty(z) = \sqrt{Q_0(c_0)/(\pi E_z)} \quad \text{as } z \to \infty
\]

Before matching with hump, many possible tendrils

After matching unique tendril
Conclusion

- Volume flux of liquid entrained from lower layer

- Scaling estimate for tendril thickness
  \[ R \sim (\mu_0 / \mu)^{1/2} H \]

- Steady tendril is thin because it has nearly merged onto an unstable hump solution

\[ Q_0 = \frac{16 \pi \mu^2 \mu_0 E^4}{(\Delta \rho g)^3} \left[ \gamma_1 \log \left( \frac{S}{\ell z \sqrt{\mu_0 / \mu}} \right) + \gamma_2 \right] \]

Shape transition at zero withdrawal rate

Supported by DOE graduate fellowship (LES) MRSEC (U. Chicago) NSF-CBET